INVESTIGATION OF COMPETENCE IN NUMERACY SKILLS AMONGST

FORM I ENTRANTS IN TANZANIA: A CASE OF SCHOOLS IN EASTERN INSPECTORATE ZONE

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**A THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN EDUCATION OF THE OPEN UNIVERSITY OF TANZANIA**

**2015**

**CERTIFICATION**

The undersigned certify that they have read and hereby recommend for acceptance by The Open University of Tanzania, a thesis titled: Investigation of Competence in Numeracy Skills amongst Form I Students in Tanzania: A Case of Schools in Eastern Inspectorate Zone, in fulfilment of the requirements for the degree of Doctor of Philosophy.

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Signature

…………………….………

Date

# 

# **DEDICATION**

This work is dedicated to my father; the late Ta Revelian L. Mwesiga, who during the time of writing the first draft of this thesis, I was besides his bed at Hyderabad Apollo Hospital in India. His anxiety to know the progress of my study was a great encouragement to accomplish this work.

# **ACKNOWLEDGEMENTS**

First, I would like to thank my Supervisors: Prof. C. K. Muganda and Prof. R. W. P. Masenge. Without their guidance, critique and encouragement, completion of this work would have not been possible. My appreciation goes especially to Prof. R. W. P. Masenge, my advisor on technical matters in mathematics. I extend my thanks to Dr. F. M. Mulengeki and other Faculty of Education staff at The Open University of Tanzania. Their assistance was very helpful.

My employer, the Ministry of Education and Vocational Training is acknowledged for granting me permission to pursue this highest degree programme. I am thankful to both the Chief School Inspector (Headquaters) and Zonal Chief Inspector (Dar es Salaam Zone) for all the assistance they provided me. I appreciate the responsive co-operation I received from heads of schools, teachers, and students in the sampled schools, as their collaboration enabled me to collect the required data.

I am very grateful to my mother; Ma Ernestina Mukalweika, my Uncle Mr. Thadeo Mtembei, my young sisters and brothers for their love, encouragement and prayers. I also, recognise the role and contributions of my wife Gerades Kokuaisa and our sons Muberwa, Bashange, Ishemo and Mugisha. The significance of their contributions is in their acceptance to take additional family chores and responsibilities, hence allowing me sufficient time to spend on my studies. They accepted reducing the time allocated for spending with either a husband or a father.

Finally, I owe much thanks to my colleagues at the Dar es Salaam Zonal School Inspectorate Office who assisted me in one way or another. However, I am solely responsible for any shortcomings that are in this work.

# **ABSTRACT**

The study has employed a descriptive research design and elements of both qualitative and quantitative methods to accomplish the fundamental goal of investigating the competence of Form I entrants in numeric skills. The focus was on identifying students’ errors on primary school mathematics, the associated misconceptions and causes. Consequently, a remedial approach was designed for intervention. The test, questionnaire and a focus-group discussion guide were employed for data collection. While errors were explored by a scrutiny of students’ work sheets, discussion guide was utilized to ascertain the conjectures of underlying misconceptions and causes. A questionnaire was used to capture views of teachers on the ability of Form I entrants in mathematics, and interventions taken. The major findings were that: most students had inadequate numeracy knowledge and skills; they committed both conceptual and procedural errors; schools had no system of identifying students’ mathematical learning difficulties for intervention; and there was no an effective remedial approach for correcting mathematical misconceptions amongst Form I entrants. The study recommends: (i) A system of testing Form I entrants on competencies in numeracy knowledge and computation skills, that are requisite for understanding secondary school mathematics. (ii) Introduction of a remedial programme to all Form I entrants by using a remedial approach developed in this study. (iii) Use of teaching strategies which allow diagnosis of learning misconceptions and remedial activities during the lesson. (iv) Immediate scoring of assignments, identification of errors and provision of feedback through extensive corrections. (v) A review of policies on teacher training and textbooks to address the issue of teaching/learning of mathematics at primary and secondary school levels.

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**ACRONYMS AND ABBREVIATIONS**

BODMAS Brackets of Division, Multiplication, Addition and Subtraction

CSEE Certificate of Secondary Education Examination

FTSEE Form Two Secondary Education Examination

INSET In Service Training

MAT Mathematical Association of Tanzania

MA-TEST Mathematics Achievement Test

MOEC Ministry of Education and Culture

MOEVT Ministry of Education and Vocational Training

NECTA National Examinations Council of Tanzania

PSLE Primary School Leaving Examination

SACMEQ Southern and Eastern Africa Consortium for Monitoring Educational Quality

SPSS Statistical Package for Social Sciences

STIP Science Teaching Innovation Programme

TC Teachers College

TIE Tanzania Institute of Education

TSS Takwimu za Shule za Sekondari

# **CHAPTER ONE**

# **1.0 GENERAL INTRODUCTION**

## 1.1 Introduction

This chapter presents the background to the problem, statement of the problem, objectives, research questions, significance, conceptual framework, limitations and delimitation of the study as well as definitions of underlying terms.

## 1.2 Background to the Problem

According to Walshaw and Anthony (2009), understanding of mathematics influence decisions making in all areas of life internationally i.e. private, social, and civil life. Thus, mathematics education is a key to increasing the post-school and citizenship opportunities of young people. Many students struggle with mathematics and find it difficult as they continually encounter difficulties in learning. It is therefore imperative that, we find out what we can do to break this pattern.

The focus of this study is on students’ acquisition of basic mathematical knowledge and skills. Showing the importance of mathematics worldwide, UNESCO (2011:10) states that:

“Mathematics is omnipresent in today’s world, notably in the technological subjects surrounding us, and in exchange and communication processes – but it is generally invisible. Hence there is a lack of awareness of the importance of developing a mathematics culture beyond basic knowledge relating to numbers, measurements and calculations. It is important that basic education removes this invisibility, especially because the needs attached to so called mathematical literacy go well beyond the needs traditionally associated with basics computational knowledge”.

Available literature indicates that students at primary and secondary education require a set of mathematical skills, usually known as “numeracy skills”. The most basic aspects of numeracy skills are Addition, Subtraction, Multiplication and Division. Knowledge of addition and subtraction is the first step in numeracy skills. These are taught from early age to encourage children to enjoy working with numbers and to understand how numbers are used in daily life situations. Ability to multiply and divide numbers is the next step. These skills are also helpers with the more advanced levels of mathematics that students will encounter during their school lives, and also into their adulthood. However, as UNESCO (2011) points out, this kind of mathematics literacy is not only the goal of mathematics education in primary schooling, but the fundamental priority. Furthermore, it is urged that the needed mathematics literacy for young people is to develop mathematical knowledge and competencies necessary for integrated and active participation in a given society, and also for adaptation to foreseeable developments.

Poor understanding of mathematics at basic education level is a world-wide problem. Research evaluations done by UNESCO internationally show that at the end of basic education, mathematics knowledge and competencies of many pupils are not at the targeted level (UNESCO, 2011). Also, the report by the International Mathematics Union (IMU, 2009) shows that, the state of Mathematics in most of African countries is affected by teaching weaknesses which begin in the primary schools where mandatory education laws like the Universal Primary Education, have caused enrolments to rise, with no consideration of required teachers both quantitatively and qualitatively. In particular, the report also pointed out that, the state of Mathematics in Tanzania and Kenya is poor especially at the primary and secondary school levels.

In Tanzania, mathematics is a compulsory subject at levels of primary and ordinary secondary education. Normally, all secondary schools in Tanzania admit Form I students who have completed the primary school education. While the Government and Community secondary schools enrol Form I entrants who have passed the Primary School Leaving Examination (PSLE), Seminaries and Non-Government schools administer an entrance examination prepared by the individual schools. This examination usually comprise of English and Mathematics components. As such, all Form I entrants admitted in secondary schools are expected to have done well in English and Mathematics.

Ordinary secondary school students in Tanzania are examined twice at national level. First they are examined at the end of Form II when they sit for the Form Two Secondary Education Examination (FTSEE). This examination is prepared and administered by the School Inspectorate Department in the Ministry of Education and Vocational Training (MOEVT) for the purpose of doing formative evaluation at this intermediate stage of O-level secondary school cycle. No certificate is awarded for that examination. Secondly, they are examined at the end of Form IV, when they sit for the Certificate in Secondary Education Examination (CSEE) that is prepared by the National Examinations Council of Tanzania (NECTA). This is a summative evaluation, for which students are awarded certificates. One could have expected good performance in Mathematics at FTSEE and consequently in the CSEE since the Form I entrants in secondary schools had passed Mathematics in the PSLE, thus perceived as having a good foundation in Mathematics. However, the students’ performance in these two types of examinations had been consistently poor for many years. For example, the national means in the Mathematics FTSEE for the examination cycles 2003- 2007 were very low, fluctuating from 14.2 to 22.8 as shown in Table 1.1.

**Table 1.1: Zones Performance in Mathematics FTSEE for Years 2003 -2007 by Means**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Zones** | **Years** | | | | | |
| **2003** | **2004** | **2005** | **2006** | **2007** | **Average mean** |
| Central | 23 | 13 | 20 | 23 | 17 | 19.2 |
| Eastern | 27 | 19 | 22 | 23 | 23 | 22.8 |
| Highlands | 24 | 18 | 18 | 21 | 18 | 19.8 |
| Lake | 25 | 11 | 16 | 21 | 18 | 18.2 |
| North Eastern | 26 | 14 | 16 | 19 | 17 | 18.4 |
| North Western | 26 | 14 | 15 | 18 | 16 | 17.8 |
| Southern | 20 | 9 | 12 | 15 | 15 | 14.2 |
| Western | 26 | 14 | 17 | 22 | 20 | 19.8 |
| **National mean** | **25** | **15** | **17** | **20** | **18** | **19.0** |

**Source**: MOEVT (2004-2008)

Also as shown in Table 1.1, the national average mean in that period of five years (2003 – 2007) was 19.0. Similarly, poor performance in mathematics had been recorded in the CSEE. For example, the failure rates in the examination cycles 1996 - 2001 ranged from 69.5% to 78% (Kitta, 2004). For this reason one may think that, the Form I entrants in Tanzania were weak in some areas of primary school mathematics, and this was resulting into learning difficulties of secondary school mathematics.

This study is therefore associating the students’ poor performance in the mathematics FTSEE with their incompetence in numeracy skills taught at primary school level. In this section the discussion on the performance of students in mathematics is basing on the PSLE, FTSEE and CSEE results. We assumed that all candidates who sat for these examinations; were taught by qualified teachers, interacted with required textbook and recommended teaching aids, covered the syllabus content adequately, and did the examinations in the same conditions. In the contrary, the said results can be considered unreliable and invalid.

Raymond (2000) contends that, numeracy skills taught in primary schools include basic concepts, algorithms, procedures, and methods of problem solving which are closely connected to concepts of mathematics topics in the primary school curriculum. The focus of this study was on the competence of Form I entrants in numeracy skills taught at primary school level in Tanzania. These skills were prerequisite for learning secondary school mathematics. Incompetence in numeracy skills is more likely to result into learning difficulties of secondary school mathematics when are not removed. Consequently they may contribute to the students’ poor performance in secondary mathematics as depicted in the FTSEE results.

Groves (2001) describes numeracy as encompassing not only mathematical concepts

and skills, but also; mathematical thinking skills, general thinking skills, problem solving strategies and a deep understanding of the context within which these concepts and skills are to be applied. In broad terms, the Department of Education Training Directorate in Australian (2009) views numeracy as the effective use of mathematics to meet the general demands of the life at school and at home, in paid work, and for participation in community and civic life. For this reason, we urge that, the Form I entrants need to have a good mastery of numeracy skills as a foundation for them to succeed in secondary school mathematics and later in life generally.

Learning difficulties are one of the main reasons for students’ failure, or for them to give up on learning mathematics if they keep making mistakes in doing mathematics. There are many causes of students’ difficulties in learning mathematics. Resnick (1982) attributed students’ learning difficulties to concepts learning, and that, difficulties in learning are often a result of failure to understand the concepts on which procedures are based. Raymond (2000) identifies three kinds of difficulties that students come up against in mathematics learning. These are temporary difficulties, recurrent difficulties and standing or challenging difficulties. It is explained that temporary difficulties occurs either due to the extent of newness for students, or due to misleading similarities to what is already known, or on the complexity of the underlying methods. Recurrent difficulties as explained by Raymond (2000) are due to the change of a context, while standing difficulties appears whenever students are faced with a proof, or some verbal problem in arithmetic, or in algebra. The author advises that, all these kinds of difficulties must be taken into account in the teaching of mathematics.

A study by Wang *et al*. (2009) on improving high school students with learning difficulties in China, revealed two types of learning difficulties namely learned helplessness and defensive attribution. The former is related to students’ confidence especially when they lack help for learning mathematics. As Slavin (2003) noted, some students fail in mathematics because of rarely getting help in their learning process.

As Xiaobao (2006) contends, the research on students’ errors and misconceptions is one way of providing a support for both teachers and students to address difficulties in learning Mathematics. Radatz (1979) classified students’ errors in terms of:

1. Language difficulties; that is, the misunderstanding of the semantics of mathematics language may cause students’ errors at the beginning of problem solving. This is because mathematics is a foreign language for students who need to know and understand mathematical concepts, symbols and vocabulary.
2. Difficulties in processing ionic and visual representation of mathematical knowledge.
3. Deficiency in requisite skills, facts and concepts; that is, students may forget or be unable to recall related information in solving problems.
4. Incorrect associations or rigidity; that is, negative transfer caused by converting the information.
5. Application of irrelevant rules or strategies.

As various researchers contend, students’ errors vary to a great extent. For example, Xiaobao (2006) describes systematic errors as one type among many different types that occur to many students over a long period, and it is relatively easy and thus possible to research. The researcher relates the cause of systematic errors to inadequate mastery of procedural and conceptual knowledge, or links between the two types of knowledge. Hiebert and Lefevre (1986) differentiate between conceptual knowledge and procedural knowledge in the domain of mathematics. They described procedural knowledge as rules, algorithms, formal language of mathematics or procedures used to solve mathematical tasks. Conceptual knowledge is thought of as connections among information, a network of mathematics facts and propositions. Basing on these descriptions of mathematics knowledge, systematic errors can often be traced to flaws in conceptual knowledge or lack of conceptual/procedural knowledge linkage.

Various researchers like Allen (2007), Young and O’shea (1981), Xioabao (2006) and Resnick (1982) view misconceptions as one of the main causes of students’ faulty algorithms and errors. According to Allen (2007), one problem that leads to serious learning difficulties in mathematics is the misconceptions students may have from previous inadequate teaching, informal thinking, or poor remembrance. Young and O’shea (1981) interpret misconceptions as students’ immature explanations of concepts which are stable and resistant to instruction.

This study concurs with Xiaobao (2006) that: if students’ misconceptions are ignored, it may lead to negative effects on their new learning, and may also reinforce original misconceptions. For this reason, it is imperative for teachers to identify students’ learning misconceptions for correcting them. As Allen (2007) and Resnick (1983) argue, students do not come to the classroom as blank slates; rather they come with informal theories which have been actively constructed from everyday experiences, although are often incomplete half-truths.

This study also supports the view raised by Kheong (1982) and IGNOU (2012) that, re-teaching a lesson will not help students who base their reasoning on strongly held misconceptions. This is because students tend to be emotionally and intellectually attached to their misconceptions, partly because they have actively constructed them and partly because they give ready methodologies for solving various problems. Thus, the present study also suggests that, learning misconceptions must be changed internally, partly through the students’ belief systems, and partly through their own cognition as recommended by Allen (2007).

In Tanzania, an attempt to prepare Form I entrants for learning secondary school subjects was done by MOEC (1994) through its project best known as *English Language Teaching Support.* The projectproduced the Baseline Manual recommended to be used in all secondary schools in Tanzania as an orientation course material for Form I entrants. The content of this manual which covers topics in Mathematics, Geography, Physics, Chemistry, Biology and History is designed to be completed within six weeks after Form I have reported at school. However, the manual aims to improve students’ ability on language demands in the respective subjects. In due respect, the manual first produced in 1994 do not address the issue of providing Form I entrants with requisite numeracy skills for learning secondary school mathematics.

This study hypothesized that, Form1 entrants need to have the required competence in numeric knowledge and skills in order to understand better the basic concepts and skills of secondary school mathematics. Based on the Basch model, the Southern and Estern Africa Consortium for Monitoring Education Quality (SACMEQ II) study by MOEC (2005) identified eight levels of competencies to assess the mastery levels of mathematical skills of Standard VI pupils. The first five levels focused on numeric skills. Numeric skills entail an ability and competence to manipulate numbers using the basic arithmetic operations of addition, subtraction, multiplication and division. The said numeric skills are *pre-numeracy; emergent numeracy; basic numeracy; beginning numeracy and competent numeracy.* These skills were taught at the primary school level in Tanzania (MOEC, 2005).

The findings in the SACMEQ II report show that: about 25% of Standard VI pupils reached the first and second levels, 35% reached the basic level, 21.4% reached the beginning level and 9.9% reached the competent level. This disappointing situation is reflected in the Mathematics PSLE statistical results compiled by NECTA. The results indicate that, for the years 1994-1998 and 2000-2002, the percentage of failures in Mathematics was 81.9% and 80.0% respectively (MOEC, 2003). This massive failure suggests that, the majority of primary school leavers had no good understanding of mathematical knowledge and skills they were taught at primary school level.

Hence, there is a need of investigating the areas in primary school mathematics where pupils have learning difficulties, types of difficulties and the underlying causes. Specifically, this study is an attempt to identify the common errors committed by Form I entrants in primary school mathematics, the underlying misconceptions and their causes. If these are identified and remedy action taken, there will be a possibility of learning secondary school mathematics successfully.

**1.3 Statement of the Problem**

Mathematics is one of the compulsory subjects taught at Ordinary secondary school level in Tanzania and elsewhere. Unlike other subjects, students’ misconceptions about mathematics in particular, affect further learning due to the hierarchy of mathematics knowledge structure (Resnick, 1982). Error patterns in students’ mathematics work suggest learning misconceptions that students may have. Thus, misconceptions are one of the main causes of students’ faulty algorithms and errors in problem solving.

Without sound understanding of basic mathematics concepts and skills, it is almost impossible for students to develop advanced thinking, and succeed in further mathematics learning. This is so for two main reasons; first, in the process of learning, students connect new information with what they already know. Second, prior knowledge is not always correct knowledge, it may entail misconceptions. As Xiaobao (2006) contends, to ignore students’ misconceptions may have negative effects on students’ new learning and can also reinforce original misconceptions. Thus, teachers need to identify students’ preconceptions or misconceptions so as to help them learn mathematics effectively and efficiently.

However, as Xiaobao (2006) observes there has been a tendency to ignore or underestimate the complexity of students’ learning difficulties in mathematics by taking teaching approaches that reject sophisticated issues of learning mathematics. In Tanzania, this is reflected in Base Line-course material prepared to orient Form I entrants in secondary school subjects including mathematics. As already explained in section 1.1 this material is not addressing the issue of students’ competence in basic mathematical knowledge and skills requisite for learning secondary school mathematics.

To address such problems, this study intended to assess the ability of Form 1 entrants in primary school mathematics; secondly, to diagnose students’ errors, misconceptions and underlying causes; thirdly to identify common errors and how to remove learning misconceptions facing Form I entrants; and finally to provide a sample of remedy approach as a guide to conducting remedial lessons.

**1.4 Objectives of the Study**

### 1.4.1 General Objective

The general objective of this study was to investigate the competence of Form 1 entrants in numeric skills in order to find the most effective remedial measures for taking action.

### 1.4.2 Specific Objectives

The study aimed at achieving the following specific objectives:

1. To find out the competence of Form I entrants in the Primary school mathematics.
2. To identify the errors committed by Form I entrants in the numeric skills learnt at primary school level as depicted in the Mathematics Achievement Test (MA-Test).
3. To identify the mathematical misconceptions which lead to the observed errors in numeric skills.
4. To conjecture the students’ misconceptions on numeric skills and their causes.
5. To design an appropriate remedial approach for removing errors committed by Form I entrants.

## 1.5 Research Questions

The study sought to answers the following research questions:

1. What is the ability of Form I entrants in mathematics as depicted by their performance in the PSLE?
2. What types of errors do Form I entrants make in numeric skills?
3. Which mathematical misconceptions lead to the observed numeric errors?
4. What are the causes of the identified misconceptions?
5. What mathematics remedial approach is appropriate for orienting Form I entrants in numeric skills?

**1.6 Significance of the Study**

The study is significant to all teachers at both primary and lower secondary education levels, and other education experts in improving the teaching and learning of mathematics in the following ways:

1. The analysis of errors made on each question item might provide information to enable the development of better teaching strategies to address the probable learning difficulties on the topics of concerned items.
2. During teaching it might be possible for teachers to alert students on errors likely to be committed when solving problems.
3. Tanzania Institute of Education (TIE) and other writers of mathematics books could use the findings in such a way that, they present and discuss common misconceptions and errors using model solutions, instead of presenting only correct material.
4. The remedial approach developed in this study may serve as a useful teacher’s guide in preparing remedial lessons for Form I orientation course, while the Ministry responsible for education is looking for ways and means of solving the problems of ineffective teaching of mathematics at primary and secondary school levels.
5. Remedial package addressing all errors identified in this study can be prepared by adapting the developed sample remedial lesson. The package may serve as useful material for orienting Form I entrants, and a training reference to the primary school teacher-trainees, since are the ones being prepared to teach mathematics at primary school level. The package should be validated and tested by the MOEVT experts before put in use.

## 1.7 Theoretical Framework

Olivier (1989) asserts that “one cannot ...discuss the matter without using some theory to explain the situation”. For this reason, we cannot say why students often commit errors in mathematics unless we make our interpretation by using a learning theory. The theoretical framework of this study is the constructivists’ perspective on learning. As opposed to the behaviourists’ perspective on learning, the constructivists believe that, concepts are not taken directly from experience, but that, a learner’s ability to learn from, and what he learns from an experience, depends on the quality of the ideas that is able to bring to that experience (Olivier, 1989). In the eyes of constructivists, children do not only interpret knowledge, but they organise and structure this knowledge into large units of interrelated concepts or ideas called *schemas*. Such schemas are important intellectual tools, stored in memory, and which can be retrieved and utilized. The author maintains that, learning basically involves the interaction between a child’s schemas and new ideas. Thus, to understand an idea means to incorporate it into an existing schema.

Interaction in the constructivists perspective involves two interrelated processes namely *assimilation* and *accommodation*. In the process of assimilation a new but recognisable idea is directly incorporated into an existing schema, and it contributes to our schemas by expanding existing concepts, and by forming new distinctions through differentiation. In the accommodation process, sometimes a new idea may be quite different from existing schemas. Here we may have a schema which is relevant, but not adequate to accommodate the new idea. In this case, it is necessary to re-construct and re-organise our schema.

Such reconstruction leaves previous knowledge intact as part or subset or special case of the new modified schema, without erasing previous knowledge. Rote learning happens when a new idea is so different from any available schema, and it becomes impossible to link it to any existing schema. Here assimilation and accommodation is impossible. In such a case, the learner creates a new “box” and tries to memorise the idea. Such rote learning is the cause of many mistakes in mathematics as pupils try to recall partly remembered and distorted rule. This study had employed constructivists’ perspective as its theoretical framework mainly because it views misconceptions as crucially important to learning and teaching, as they form part of a pupil’s conceptual structure that will interact with new concepts, and influence new learning.

**1.8 Conceptual Framework**

This study focuses on competence of students in numeric skills taught at a level of primary school; and ways to remedy learning difficulties that the Form I entrants may have on primary school mathematics. For effective learning of secondary school mathematics, secondary school beginners need to have a good mastery of numeric skills as already discussed in section 1.2. This is because; students are likely to face difficulties in making connections of concepts of secondary school mathematics with what they already know from primary school mathematics. Figure 1.1 conceptualizes the relationships of concepts involved in the research problem of this study.

Students’ learning difficulties in mathematics are mainly due to poor mastery of mathematical conceptual and procedural knowledge in solving a problem. As such, the poorly mastered knowledge leads to learning misconceptions, which result into committing errors when solving a problem. Therefore, it is very important to diagnose the types of students’ errors on primary school mathematics, their causes, and how to address them for providing a good foundation of learning further mathematics beyond primary school level.

**Figure 1.1: Remediation of Misconceptions and Errors in Numeric Skills**

**Remedial approach**

* Formulating remedial lesson objectives
* Identifying required conceptual and procedural knowledge
* Organizing teaching aids
* Devising group activities
* Delivering remedial instructions
* Providing worked examples
* Providing group activities
* Summarizing main points
* Giving homework
* Doing assessment and evaluation
* Feedback
* Reflection

**Conceptual errors**

**Procedural errors**

**Diagnosis of common errors**

* Achievement test
* Analysis of answers of each test item
* Identification of common errors

**Learning misconceptions**

* Due to application of procedures/rules
* Due understanding of concepts

**Source:** Adapted from Li (2006)

**Source:** Adapted from Xiaobao (2006)

**Source:** Adapted from Xiaobao (2006)

In the context of mathematics, to diagnosis refers to the identification of weaknesses and deficiencies of students in the subject. As Kheong (1982) argues, administering an achievement test is one way of diagnosing students’ difficulties in mathematics. Further more, the author points out that the construction of achievement test meant for the purpose of diagnosis should consider the following aspects: cover all skills in areas of students’ weaknesses, questions on each skill should aim to reveal students’ weaknesses when scored incorrectly, and there should be several question items on each skill to ensure reliability of a test. Through examining the answer scripts of a test given to students, teachers are able to discover varieties of incorrect ways the students used in working out solutions of problems. In this study the MA-Test was composed by the researcher to suit the diagnosis requirement, administered to Form I entrants, scored and results analysed to identify common errors committed by the students.

Students make many types of errors in mathematics including systematic errors and others due to inattention. Systematic errors are usually a sequence of students’ misconceptions on conceptual and procedural knowledge. The focus of this study is on conceptual and procedural systematic errors which emerged as common to a number of students attempted the MA-test. Nevertheless, all types of errors need to be addressed. The importance of errors is that, they reflect students’ understanding of a concept, problem or procedure of solving a problem (Sarwadi and Shahrill, 2014). Procedural errors are due to lack or inadequate understanding of procedural mathematics knowledge. This knowledge is on skills and step-by-step procedures without explicit reference to mathematical ideas. Students committing procedural errors do not have an understanding of why or how procedure works; and do not recognize the importance of applying, and computing the procedure correctly (Elbrink, 2008).

Conceptual errors result from lack or poor understanding of conceptual mathematical knowledge. Hope (2006) refers conceptual mathematics understanding as a knowledge that involves clear and systematic understanding of fundamental basic points behind the algorithms performed in mathematics. As Hull and Miles (2010) argue, students demonstrate conceptual understanding in mathematics when they provide evidence that they can generate examples of concepts; use and interrelate varied representations of concepts; identify and apply principles; know and apply facts and definitions; and apply the signs, symbols, and terms used to represent concepts. Conceptual understanding also allows a student to apply and possibly adapt some acquired mathematical ideas to new situations. Thus, to avoid procedural and conceptual errors in mathematics, both procedural and conceptual learning are necessary with a consideration that, procedural learning needs to be coupled to conceptual learning and to real-life applications.

When students make some generalizations which are not correct, they create misconceptions. A misconception is not wrong thinking. Instead, it stands as a premature concept or a local generalization that the student has made. Thus, it is very difficult to avoid misconceptions during learning since are formed when students systematically use incorrect rules, or use correct rules beyond their proper domain of application (Askew and William, 1995). The relationship between errors and misconceptions is that, the latter give rise to the former. For this reason, teachers need to be alerted to misconceptions and error patterns amongst the students. In this study an attempt was done to identify the misconceptions and error patterns in students’ work sheets on the administered MA-Test, and to think about why they have used the procedures they did. The ultimate aim was to design a remedial approach for removing the identified errors and underlying misconceptions.

Respondents for the study were Form I entrants and mathematics teachers from the nine selected secondary schools in the Eastern School Inspectorate Zone. The zone, schools and teachers were purposively selected; while random selection was employed to obtain a sample of students. MA-Test, questionnaire and focus-group discussion guide were employed for data collection. Students’ work sheets were scored, followed by identification of committed common errors, underlying misconceptions and causes. These three types of data were tabulated for each question item. All collected data including those from teachers’ questionnaire were subjected to content analysis.

Remedial approaches are recommended when students lack a certain skill or ability, but still possessing the ability to master that skill or ability. Remedial instructions intends to enable students have a good mastery of foundational skills required for them to make progress with more advanced skills and concepts. Some remedial techniques as identified by Elbrink (2008), Karibasappa *et al*. (2008) and Lawrence *et al*. (2010) include breaking tasks down into smaller portions, re-teaching skills, introducing the concept before the procedure, real-word applications, reflection and self assessment, and formulating group learning activities. The most important point is that, an effective remediation approach must constitute instructional strategies that promote thoughtful, active learning, and connections to early concepts. In this study, a remedial approach for addressing the identified common errors was suggested together with a sample of a remedial lesson plan.

**1.9 Limitation of the Study**

There were several factors contributing to students’ incompetence in mathematics at secondary school level. However, this study was limited to students’ errors and misconceptions in numeracy skills as the most crucial factor among others. This was because the philosophy in which the research problem was conceived acknowledges the view by the constructivist perspective, that misconceptions (causing errors) are crucially important to learning and teaching. And that, they form part of a pupil’s conceptual structure that will interact with new concepts, and influence new learning (Olievier, 1989). As such, the data and discussion are limited to errors committed by students in numeric skills and associated misconceptions, rather than other substantial factors affecting the teaching and learning process of the subject. Secondly, unlike other subjects, students’ misconceptions about mathematics particularly affect further learning due to the hierarchy of mathematics knowledge structure. Hence it is necessary to change students’ misconceptions as they arise, and before they are introduced to new concepts.

**1.10 Delimitations of the Study**

There were eight School Inspectorate Zones in Tanzania mainland. Data for the study was collected from only one zone, the Eastern School Inspectorate Zone. The Zone had a total of 532 registered secondary schools in 15 districts of Dar es Salaam, Morogoro and Coast regions. In an ideal situation it would have been desirable to cover all the schools in the Zone, and have a more diverse target population of students in terms of regional differences. However, the scope of the study was narrowed down to only nine schools. This in turn reduced the coverage area of the study. Nevertheless, the findings were generalized to characterize all Form I entrants in the country since the selection of schools had considered the regional differences, that is, urban, semi urban and rural areas.

In this study the respondents were drawn only from a group of primary school leavers who passed either the PSLE or the school based entrance examination. This was purposely done for two main reasons. First, those who failed in the said examinations were not in good position to write solutions of the questions in the competence test, and secondly it was possible to take remedial action to the group of those passed the PSLE since they were in schools unlike the group of those not selected.

## 1.11 Definitions of Underlying Terms

Fraenkel and Wallen (2000) suggest the use of operational definitions by specifying the actions or operations necessary to characterize the terms. This is done for the purpose of clarifying meanings and therefore making the message communicated by the sender to be perceived by the receiver in the intended meaning or with minimum ambiguity. As such, the terms competence, numeracy, computational errors, learning misconceptions and conjectures as used in this study are operationally defined as follows:

**1.11.1 Competence**

Students’ competence or proficiency is referred to the attainment of academic expectations in a subject area, or grade level, basing on the set learning standards. The general goal of learning is to ensure that students are acquiring the knowledge and skills that are deemed to be essential to success in school, higher education, careers, and adult life. Students are therefore expected to demonstrate that, they have learned the expected knowledge and skills as they progress from one stage of education to another. When students fail to meet the expected learning standards, efforts should be done to provide them with an additional instruction, practice, time, and academic support to help them achieve competency or meet the expected standards.

### 1.11.2 Numeracy

In broad terms Groves (2001) defines numeracy as the ability to use mathematical concepts and skills efficiently to meet general demands of day-to-day life and to make sense of the world. General demands as discussed by the author include the following: The use of a combination of numerical, spatial, graphical, statistical, and algebraic mathematics thinking and strategies; To use mathematics to interpret information or to solve practical problems; To choose mathematical skills and strategies that make sense in particular circumstances; To apply mathematics knowledge appropriately in a range of contexts where mathematical reasoning is required and; To monitor and evaluate the effectiveness of actions and judge what is reasonable.

### 1.11.3 Errors

Students make mistakes in solving mathematical problems either due to carelessness, poor or no understanding at all, confusing different concepts or failing to apply rules despite being taught. Such mistakes, frequently done by many students, are referred to in this study as errors. According to Resnick (1982) these errors are caused by difficulties in learning, which leads to failure to understand the concepts on which operations are based.

**1.11.4 Misconceptions**

The literature shows that serious learning difficulties in mathematics are caused by misconceptions that students have, either are due to inadequate teaching, informal thinking, or poor memory. The Encarta online dictionary defines a misconception as a mistaken idea or view resulting from a misunderstanding of something. It is emphasized that a misconception does not exist independently, but is subject to a certain existing conceptual framework. In this regard, Mestre (1987) views students’ misconceptions as informal theories which are often incomplete or half-truths. Students tend to be emotionally and intellectually attached to their misconceptions, partly because they have actively constructed them and partly because they give readymade methodologies for solving various problems. As Mestre (1987) contends, removing students’ misconceptions cannot be done by merely informing or lecturing them on their misconceptions. The misconceptions must be changed internally partly through the students’ belief systems and partly through their own cognition by providing them with counterexamples to their misconceptions, especially self-discovered counterexamples (Allen, 2007).

**1.11.5 Conjectures**

A conjecture is a good guess or an idea about a pattern. According to Cooney *et al*. (1975), a conjecture in the context of mathematics is a conclusion or proposition which appears to be correct based on incomplete information, but for which no proof has been found. A conjecture can be thought of as the mathematicians’ way of saying ‘I believe that this is true, but I have no proof yet’ (Cooney, et al., 1975).

## 1.12 Organization of the Study

This thesis is organized in five chapters. The first chapter is an introduction, where the background and the statement of the problem are presented. Stated in this chapter are also the research objectives, research questions, significance of the study, theoretical framework, conceptual framework, limitations, delimitations and definitions of underlying terms.

Chapter two presents a review of related literature organized under eleven sections on the basis of the objectives of this study. These are: Introduction, How students learn mathematics, Basic causes of Students’ Learning Difficulties, Mathematical errors and misconceptions in numeric skills, Diagnosis in the context of Mathematics, Methods of diagnosing learning difficulties and Remediation in the context of mathematics. This review also examines different studies conducted inside and outside the country pertaining to problems of teaching and learning mathematics, and how to overcome them. Findings of these studies help to portray the knowledge gap which the present study attempts to bridge.

Chapter three is on the methodology employed in the study. This study employed elements of both qualitative and quantitative methods for which a descriptive design was used to do a careful and complete observation of each and every aspect under consideration, as well as to draw inferences from data generalizations. The chapter also presents the area of study; the population, sample size and sampling procedure; types and sources of data; data analysis plan; reliability, internal and external validity; and ethnical dimensions for the study.

Chapter four is the presentation, analysis and discussion of the findings based on the research objectives and questions about the ability of Form I entrants in mathematics, as well as types of errors and misconceptions they commit in numeric computation skills. These respond to the first four specific objectives of the study. The first objective sought to investigate the performance of Form I entrants in the PSLE- mathematics subject and the second objective was to investigate the common errors committed by the said entrants on primary school mathematics as revealed from the MA-Test. The third and fourth objectives sought to conjecture the associated misconceptions and underlying causes which lead to the committed errors. The fourth objective was to identify conjectures for the possible causes of the misconceptions. The last objective sought to develop an appropriate format of remedial approach, and a provided a sample of a remedial lesson for eliminating students’ errors and misconceptions.

Chapter five gives a summary of the study, conclusions and recommendations consequent to the analysis and discussion in Chapter four.

# **1.13 Concluding Remarks**

This chapter has presented the general contemporary issues of mathematical literacy worldwide. A connection was made to local perspectives and numeracy in particular by linking poor performance of students at secondary school level with the problem of inadequate numeracy skills among Form I entrants in Tanzania. The chapter also pointed out that, the efforts made by various education stakeholders in the country to solve the problem haven’t shown any significant improvement in mathematics students’ performance. Furthermore, the chapter explains that no effort has been done to investigate the numeracy competence of primary school leavers selected for secondary education in Tanzania. The next chapter presents the review of literature related to the problem.

**CHAPTER TWO**

# **2.0 REVIEW OF RELATED LITERATURE**

## 2.1 Introduction

This chapter reviews the literature relating to students’ performance on mathematics and the associated learning difficulties. The review also examines the concepts of errors and misconceptions in numeric computations as well as the concepts of diagnosis and remediation in the context of teaching and learning mathematics. Finally, the chapter identifies the knowledge gap and type of data and information this study requires.

## 2.2 How Students Learn Mathematics

This study embrace the belief by Walshaw and Anthony (2009) that mathematics pedagogy must focus on optimizing a range of desirable academic outcomes that include conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning. Basing on various research finding, the authors discuss some effective pedagogical approaches of improving students’ achievement in mathematics. Some of important approaches drawn from the authors include:

i. Promoting students’ relationships focusing on mathematics.

ii. Building students’ positive attitude towards mathematics.

iii. Promoting the habit of working independently and collaboratively.

iv. Building on existing proficiencies, interests, and experiences.

v. Providing tasks that stimulate original thinking of students. .

vi.Using students’ misconceptions and errors as important building blocks in learning mathematics.

vii.Assigningchallenging tasks to students.

viii. Making connections of apparently separate mathematical ideas.

ix. Promoting the habit of doing self and peer assessment among students.

x. Observing students’ understanding of mathematical language.

xi. Utilizing tools in teaching to stimulate students’ thinking.

Regarding *Using* *students’ misconceptions and errors as important building blocks in learning mathematics,* it is contended *that* errors arise from incorrect interpretations of mathematical ideas that represent the learner’s attempts to create meaning. Such incorrect interpretations should not be regarded as wrong thinking. Instead, teachers should view them as a necessary stage in a students’ conceptual development. There are many ways in which teachers can provide opportunities for students to learn from their errors. One is to organize discussion that focuses students’ attention on difficulties that have surfaced. Another is to ask students to share their interpretations or solution strategies so that they can compare and re-evaluate their thinking. Yet another is to pose questions that create tensions that need to be resolved. For example, confronted with the division misconception just referred to, a teacher could ask students to investigate the difference between 10 ÷ 2, 2 ÷ 10, and 10 ÷ 0.2 using diagrams, pictures, or number stories.

**2.3 Basic Causes of Students’ Learning Difficulties**

Cooney *et al*. (1975) identify five broad factors or causes of students’ difficulties in learning. These include physiological, social, emotional, intellectual and pedagogical factors.

*Physiological factors;* Defects in vision or hearing, lack of enough food and drug abuse are categorized as physiological factors. Each of these adversely affects learning. For example, typical classroom learning requires extensive use of vision and hearing, a hungry student cannot pay enough attention to lesson instructions and assignments; severely malnourished students may have their mental activity impaired, and students taking drugs become inattentive. However, these physiological causes of student difficulties are usually beyond remedy by the classroom teacher.

*Social factors;* Motivation at home, atmosphere of the class and relationship with peers are among social factors, which may militate against doing well.

*Emotional factors;* A student who had been doing badly in mathematics in the lower levels of education may develop fear or hatred of the subject. This is an example of emotional factor, which is likely to affect junior students in secondary schools.

*Intellectual factors;* Students with mental disabilities cannot adequately comprehend what is being taught as ordinary students do; they do not readily retain it, and cannot apply it in the problem solving.

*Pedagogical factors;* Pedagogical factors have a lot to do with how readily students learn. Pedagogical factors include teachers’ failure to apply the principles set forth in the textbook, little or no attention to motivation, inadequate feedback and little or no feedback to facilitate decisions as to whether students have comprehended what is being taught. Pedagogical factors are the ones contributing to poor teaching and hence poor understanding of the subject content.

A good mastery of primary school mathematics is a necessary precondition for successful learning of mathematics at secondary school level. Secondary schools in Tanzania recruit Form I entrants from different primary schools of which differs in the extent of using effective pedagogical approach in teaching. As such we should expect each secondary school to have a cohort of Form I entrants with different abilities in mathematics. For this reason one may anticipate that not all Form I entrants have readiness of learning secondary mathematics. These need to be identified and assisted before being introduced to secondary school mathematics. In Tanzania little is known about readiness of Form I entrants to learn secondary mathematics. This readiness can be assessed through observing their competence in numeric skills that are taught at primary education level.

## 2.4 Mathematical Errors and Misconceptions in Numeric Skills

As pointed out by Muijs and Revnolds (2000), logic is particularly important to mathematics learning since the basic numerical operations such as counting rely on logic. For example, a child may deduce that 3 > 1 given that 3 > 2 and 2 > 1. The authors add that, logic also involves learning a set of axiom that is essential for mastering mathematical concepts and techniques. However, children usually develop misconceptions about the meaning of some mathematical concepts (Muijs and Revnolds, 2000).

In his book titled ‘*Teaching Primary Mathematics’*, Suffolk (2004) argues that many misconceptions which pupils develop in the area of numbers are related to poor understanding of place value. Also that, the concept of negative numbers is difficult to comprehend because of overemphasizing the idea that a number represents a set of things. As Khalid and Baderudin (2011) observe, students have also a difficulty in making sense of the ways in which positive and negative integers are manipulated. For example, why do two consecutive minus signs form a plus sign, say in 4 – (–3) = +7. To the teacher, this implies the necessity of letting students explain how they came to their answers, whether right or wrong so as to correct wrong answers. This is important in mathematics, because a correct final result sometimes can emerge from incorrect method.



The following are some examples given on this aspect as revealed from literature.

**Student error:** Reading2008 as “two hundred and eight”.

*Associated misconception:* The pupil focuses on the zeros as being the fundamentally important digits in the numbers.

*Conjecture*: Poor understanding of place value of digits.

**Student error:** Believing that a number is larger because it has more decimal digits. For example, 3.125 > 3.14

*Associated misconception*:125 is bigger than 14

*Conjecture*: The pupil has no understanding of place value of digits after the

decimal point.

##### Student error: ÷ =



##### *Associated misconception*: Believing that the digits in the numerators are divided separately and the same for denominators.

##### *Conjecture:* The pupil knows how to divide whole numbers but not fractions.

Loh (1991) cited in Seng (2010) observes that, secondary school students face both, the problem of understanding and representing the algebraic problem. These problems lead students to commit errors in determining the steps in solving algebraic problems. Aguele, Omo-Ojugu and Imhanlahimi (2010) mention process errors as among the factors contributing to poor performance of students in mathematics. Process errors as defined by the researcher are those committed by students while carrying out mathematical operations due to violation or wrong use of process skills. Categories of process errors include conceptual, logical, translation and application errors as cited in Aguele *et al*. (2010).

Liebenberg (1997) cited in Seng (2010) investigates errors made by students in simplifying very elementary algebraic expressions. The study revealed three error patterns, namely; misinterpretation of symbolic notation, difficulty with the concept of subtraction and difficulty with integers. Sakpakornkan and Harries (2003) explored pupils’ processes of thinking in simplifying algebraic terms. The errors observed were due to difficulties in dealing with negative signs and multiplying out brackets. According to Hall (2002) the most recorded errors in solving linear equations were on transposition, switching addends and division which accounted for almost three quarters of the total number of errors observed. It was also revealed that, some students committed errors on solving algebraic problems due to application of procedures which were not taught in class (Demby, 1997).

Fajemidagba (1986) asserts that, errors committed in mathematics word problems are due to inadequate understanding of concepts featuring in mathematics word expression, and ability to choose the appropriate process of solving the problem. Research findings by Salman (2002) revealed that word problem solving in mathematics demand high level of cognitive ability to translate mathematical expressions in the absence of concrete materials. As Sarwardi and Shahrill (2014) argue, the identification of errors in mathematics is an important first step for remedial or corrective instruction. There is little evidence in Tanzania to suggest that the types of errors and underlying misconceptions that Form I entrants may have in mathematics are known.

## 2.5 Diagnosis in the Context of Mathematics

High failure rates in mathematics demand the teacher to find out which learning difficulties and operational skills students typically manifest in learning. This requires methods of diagnosing students’ problems and strategies to conduct remedial work. This section explores the concepts of diagnosis and remediation as applied in teaching and learning.

The basic purpose of diagnosis is to locate weaknesses and to determine their cause (Indra Gandhi National Open University [IGNOU], 2012). In education, diagnosis is described as the use of laid down technical procedures to identify specific learning and instructional difficulties, and where possible, to determine the root causes. Cooney *et al.* (1975) contend that the teacher has to do as much as what a doctor does in conducting diagnosis, and prescribing treatment. However, while in medical diagnosis, there are instruments to make exact and objective observation, in education there are few objective measuring instruments capable of rendering reasonable diagnosis (IGNOU, 2012). For this reason the diagnosis of difficulties in learning is done by use of properly selected diagnostic tests. These tests should contain a cross-section of test items that reflect the various aspects of achievement which the student may possess. IGNOU (2012) gives two reasons of using diagnostic tests. The first reason is to locate areas in which additional instruction is required or in which teaching methods have to be improved and the second reason is to furnish continuous specific information in order that learning activities may be most productive of the desirable outcomes. When a students’ understanding of a certain concept or a certain mathematical skill is poor, a teacher should identify the problem so that subsequent teaching can be conducted to avoid the existing problem. The teacher therefore has to plan for re-teaching or remediation.

## 2.6 Methods of Diagnosing Learning Difficulties

There is a need for making proper diagnosis and remediation if the failure rates in mathematics among pupils are to be reduced. This need demands teachers to be familiar with effective methods of diagnosis before doing a remedial task to assist students with learning difficulties. Kheong (1982) discusses two methods of doing diagnosis.

The first method is the ‘*classroom observation method’* for which the teacher gives a class an assignment and goes around examining the work done by the students. The aim is to collect data on the different incorrect ways the students used in solving problems. This method requires the teacher to gather information about common errors committed by students and causes. As pointed out by Kheong (1982) the teacher has to note three things when making an observation in the classroom. First, he/she has to accept student’s answers whether correct or incorrect. Secondly, the teacher should not make any comments when observing students in order not to influence their work, and thirdly he/she should not attempt to correct students’ mistakes while observing, since the aim is to collect all their errors and mistakes.

The second method suggested by Kheong (1982) is the ‘*Formal testing diagnosis* *method*’ which is based on the analysis of the answers of specifically constructed test items. It is stressed that the test items must cover all the basic skills of computation; questions on each of these skills have to be constructed in such a way that their incorrect responses can reveal students’ weaknesses; questions are constructed according to their associated objectives, and that there should be four to five questions for each objective. This is to ensure that an error committed is not due to carelessness. This method is more systematic though it can leave some errors undetected. For this reason it is recommended to use both methods.

The first method concurs with the three steps recommended by the IGNOU (2012) in process of educational diagnosis: First, identifying students having trouble in learning or those in need of help; second, locating the errors or learning difficulties, and third, discovering the causal factors of slow learning. What follows after identifying the areas of difficult learning is for the teacher to intervene. As previously explained in section 1.2, a selection criterion of Form I entrants in some seminaries and non-government secondary schools in Tanzania is to pass the school-based entrance examination at a pass mark fixed by a school. Unfortunately, this examination with a component of mathematics, do not serve the purpose of diagnosing areas of students’ difficulties in mathematics for intervention. This partly suggests that there is no emphasis of employing diagnostic teaching approach in the teaching and learning of mathematics. This study employed a diagnostic test to identify errors committed by Form I entrants in numeric skills.

**2.7 Remediation in the Context of Mathematics**

As argued by the IGNOU (2012), the teacher has to prepare instructional material for remediation by adopting different teaching methodologies to meet the individual or group learning needs. Remediation as described by Kheong (1982) is a careful effort to re-teach successfully what was not well taught, or not well learned during the initial teaching**.** Remediation therefore, is a step to address any deficiencies in students learning for the purpose of eliminating them before teaching the next concept or skill. This is very important because mathematics concepts build upon each other.

Furthermore, Kheong (1982) and IGNOU (2012) assert that, instructions for remediation have to differ from the original instructions to ensure that better learning occurs during re-teaching. Effective remediation requires a teacher to first identify the concepts and procedures to be re-taught, then describing appropriate instructional strategies. The teacher must have wide knowledge of several teaching methods relating to the concerned concepts or skills in order to select the appropriate remedial strategy for the student. As Laurence, Matthew and Edward (2010) argue, the teacher should observe that the selected remedial strategy is at a level which can be understood by the student and that, re-teaching of basic concepts or procedures have to be followed by more practice of the skill.

Cooney *et al.* (1975) believe that, the causes of learning difficulties in mathematics require different remediation for individual students. In case the cause is that, students do not know certain concepts or skills; those will have to be taught by following the five steps. These steps are *introduction, assertion, interpretation, justification* and *practice*. In the *introduction* step the teacherhas tofocus students’ attention on the concept or skill by describing briefly what it entails in order for students to be aware of the instructional goals.

It is also noted that, motivating students by pointing out the utility of knowledge or skill to be learned, can be effective at times other than the introduction portions of lessons. In the assertionstep, a teacher states the fact(s) or principles that are useful for doing something for the purpose of giving general advice on what to do, how to do it and perhaps the sequence in which the various steps are to be performed. In order to help weak students, the teacher is urged to do two things; first to teach a generalization and then to form one or more principles from it in order to make clear what the learner should do. In the step of *interpretation* the teacher has to take action of reviewing the meaning of terms in the instructional materials and in generalizations, as well as reviewing tasks that are prerequisite to acquisition of a desired skill.

At the *justification* stepthe teacher should enable students to determine the correctness of their answers basing on the instructional materials. This will give them a sense of knowing if the method works, and that, it can be proved. One of the advantages of making justification is to arouse interest among students who are less interested in mathematics. The nature of mathematics demand students to do practice. *Practice* is therefore a significant step in order to develop students’ ability to complete a mathematical task with speed and accuracy. It is emphasised that an effective practice must go together with both reinforcement and feedback.

Dowker (2004) asserts that if students’ early difficulties in mathematics are not remediated, they are likely to result in very severe difficulties with those more advanced topics. For this reason, interventions should take place much earlier before the Form I entrants are introduced to secondary school mathematical topics. In the words of Dowker (ibid.), this is “partly because mathematical difficulties can affect performance in other aspects of the curriculum, and partly to prevent the development of negative attitudes and mathematics anxiety” (p.iii). The planning of interventions should consider the fact that, ability in numeric skills is made up of many components, ranging from conceptual knowledge of subject content to ability of applying mathematical rules and procedures in solving mathematical problems. Dowker (2004) asserts that, interventions that focus on the particular components with which an individual learner or a group of learners have difficulties are likely to be effective. As earlier noted, the purpose of the Baseline Manual recommended for orienting Form I entrants in all secondary schools in Tanzania; do not serve the purpose of assisting Form I entrants who have no good mastery of mathematical knowledge and skills taught at primary school level. As such, there was a need of designing an appropriate remedial approach for orienting Form I entrants in mathematics knowledge and skills, that are requisite for learning secondary school mathematics.

## 2.8 Studies Done Outside Tanzania

A number of studies have been conducted on the teaching and learning mathematics. In this section we present some studies that have been conducted outside Tanzania.

Nard and Steward (2003) conducted a one year study in Norfolk, U.K to investigate students’ disaffection in secondary mathematics. Seventy pupils aged 13- 14 years were involved in the study. Through extensive classroom observations and interviews, a profile of negative perceptions regarding secondary school mathematics was constructed. Students’ view of mathematics and learning fall under two categories as follows:

*Perception of mathematics*

1. Tedious and irrelevant body of isolated, non-transferable skills, the learning of which offers little opportunity for activity.
2. A subject of memorizing and imitating of correct procedures given by the teacher in test and examinations.
3. Difficult and elitist subject that exposes the weakness of the intelligence of any individual who engages in it.

*Perception of learning*

1. The use of symbolism alienating.
2. A rote-learning activity that involves application or use of rigid rules which lead to unique methods for obtaining answers to problems.

The outcome of those perceptions is that, the lower their mathematical confidence is the less willing were students to engage with mathematics.

Xiaobao (2006) conducted a study in Texas and Delaware States in the USA to find out why so many students fail in mathematics and why mathematics is so difficult for many students. The study, which involved 317 students in Grade VI and VII, focused on the nature of students learning of basic concepts by analysing their errors and misconceptions in solving problems on the concepts of variables, equations and functions. The causes of students’ understanding were studied by comparing high-achieving and low achieving students’ understanding of the three concepts at the object (structural) or process (operational) levels.

The findings revealed that the relationship between students’ misconceptions and objective-process thinking explained why some misconceptions were particularly difficult to change. When students have understood a concept as a process with misconception, that misconception was particularly hard to change. For example, the misconception of ‘equal sign’ was the misunderstanding of either side of equation as a process rather than as an object. By considering the misconceptions found, the study concluded that, the use of re-teaching as a remediation approach may have negative effects on students’ understanding of mathematics if no significant change in the curriculum is made.

A study by Lawrence *et al*. (2010) sought to determine the effectiveness of selected teaching strategies in remediation of process errors committed by students in mathematics in senior secondary schools in Nigeria. A total of 207 students from six schools were involved. The diagnosis test in mathematics was used to collect data. The findings revealed that the direct instruction approach was a more effective strategy for the remediation of process errors committed by students in mathematics. They defined the direct instruction approach as a basic practice strategy incorporating stimulus, control, reinforcement, and modelling.

Montague, Woodward and Bryant (1993) point out seven steps of the lesson procedure for the direct instruction strategy. These involve: review of previous prerequisite learning; clear statements of lesson objectives; presentation of new material; modelling of skills; guided practice; brief review of steps used in performing the skill, and independent strategy. In adherence to the remedial programme, the study recommended the appropriate use of instructional strategies, on-going remediation during normal class instruction and provision of enough and properly supervised practice activities during the mathematics lesson.

A study in India by Karibasappa, Surendranath, Nishanimut and Padakannaya (2008) focused on the need for remedial teaching for students with mathematical disabilities, and to illustrate the effectiveness of such remedial teaching. Seventeen students with mathematical disabilities were subjected to a remedial teaching programme. The programme was not conducted to a control group of 17 students who also had mathematical disabilities. Both groups were tested in mathematics skills at the beginning and at the end of one academic year. The results showed that, the students who underwent remedial teaching demonstrated a significant improvement in pre-operational and operational domains of mathematical skills. The remedial teaching method which was used in the study employed five steps as follows:

1. Error analysis for which errors committed by students are tabulated and the teacher is to find the precise area where the student’s level of competence in a specific skill is low.
2. Development of conceptual base for which explicit examples are given to illustrate the principles and mathematical operations.
3. Developing the related language component,
4. Mastery of learning and instructional techniques for which the conceptual learning is to be done first followed by simple activities that best reinforce the concepts.
5. Teaching of fundamental mathematics which also involves teaching pre-operational and operational skills.

Seng (2010) conducted a study in Malaysia in which 40 items were administered to 265 Form II male students in simplifying algebraic expressions. Thirty students among them were selected from high, medium and low ability categories for the interview which aimed at the justification of the procedures used by the students to obtain their answers. This was aimed to have an insight into students’ reasoning which cannot be obtained from their working. The study identified 12 types of errors committed by high, medium and low ability students. The errors were considered to be the result of interference in learning; difficulty in operating with negative integers; misconceptions of algebraic expressions and incorrect application of rules. Among the errors found were:

1. Misinterpretation of the exponent 2 involving a variable such as in *a2*. Students thought that *3a2*was the same as *32a*
2. Incorrect simplification of the term *7a2*. The students thought that *7a2* is equivalent to 14*a* in its simplified form*.*
3. Detachment of a negative sign from the coefficient of a term. Students neglected the negative sign when performing operations and lastly to put it back in the solution, as in – 3*y* + 4*y* = –7*y*. In this example the operation of addition was performed without considering the negative sign attached to 3*y,* and then the negative sign was inserted in the answer.
4. Misinterpretation of symbolic notation where an invisible numerical coefficient such as 1 in front of ‘a’is regarded as ‘nothing. To them, “nothing” is interpreted as 0. For this reason, students incorrectly, did – 8*a* – *a* = – 8*a* – 0*a* = – 8*a.*
5. Combining error: for which students combined unlike algebraic terms such as 4*x* + 3 to obtain 7*x*. Consequently, they applied BODMAS rule (perform operation in brackets first) to perform 2(4*x* + 3) = 2(7*x*).
6. Error in subtracting integers: Students used the concept of subtraction in arithmetic “Larger value minus smaller value” to reverse the order of subtraction, as in 3*a* – 6*a* in which they did 6*a* minus 3*a* to obtain 3*a.* They also misapplied the rule “negative times negative is positive” in simplifying algebraic expressions involving two successive ‘–ve’ signs.



1. Distributive error in expanding brackets: At first, students multiplied the first algebraic term by the pre-multiplier and ignored the second algebraic term as in 2(4*x* + 3) = 8*x* + 3. Second, they multiplied the terms in the second bracket by the pre-multiplier of the first bracket as in 2(3*x* + 2) + (3 + 4*x*) =

6*x* + 4 + 6 + 8*x*

viii. Error in multiplication of variables: They applied the rules of addition or multiplication in simplifying *a × a,* and *2a × a*. They multiplied or added the coefficients of *a,* and copied the variables with no change. For example *a* × *a* = 2*a*, *a* × *a* = *a, and* 2*a* × *a* = 3*a*, 2a× *a* = 2*a*. They also multiplied the coefficient of a variable by a constant, as in *2a + a +* 15 *= a + a* + 30

ix. Error in the order of operation: They did this, when the addition sign appeared before the multiplication sign as in *a +* 2*a×* 5 *=* 3*a×* 5 *=* 15*a.*

x. Error of negative pre-multiplier: Students did not change the sign of the second algebraic term after multiplying out the brackets as in 3*(*6*a –* 7*) =*

*–*18*a –* 21

xi. Error in applying ‘times tables’. This error occurred because the students were not conversant with the ‘1 to 12 multiplication tables’.

xii. Errors in multiplication involving a negative multipliers: These appeared when students multiplied out negative pre-multipliers in the expansion of brackets as in –5(*a* + 3) = –5*a* +15. This happened because the students thought that the negative sign attached to a multiplier is applicable only once with the first term in the brackets, that is – 5 × *a*.



An investigation by Hall (2002) which involved 246 pupils, revealed six types of errors committed by pupils when solving simple linear equations. Data for the study was collected from the final examination. Errors on transposing, switching addends and division accounted for approximately three quarters of the total number of errors noted. According to Hall (2002), transposing and switching of addends falls under structural errors while division errors were procedural. The study also pointed out that transposing errors were due to oversimplification of the transposition process and switching of addends errors appeared more frequently in ‘algebraic’ than in ‘arithmetical’ expressions. Since the process of solving equations involves arithmetic skills, the study highlighted the importance of pupils to posses the required basic skills, such as division.

Khalid and Baderudin (2011) conducted an exploratory study to compare the effectiveness of the Jar Model teaching approach with the approach of using a number line model on the improvement of students’ understanding of operations on integers. In the Jar Model, the positive or negative elements or both are put in a jar. The positive elements cancel the negative elements and, the positive or negative elements put separately in the jar, they accumulate. The study sought to answer two questions:

1. What pre-existing knowledge do the students generally have about integers?
2. To what extent does the Jar model strategy used in the intervention enhance the students’ performance on operations with integers?

Both qualitative and quantitative methods were used to collect data from Form I (Grade 7) students in one government secondary school in Brunei. Previously, the students had already been taught operations with integers using the number line model. After being pre-tested, they were re-taught during a four weeks’ intervention strategy by using the Jar Model, and then post tested. Both teachers and students were interviewed. Pre-test results showed that students got mixed up with subtraction and negative signs in a problem. The confusion caused some errors such as:

1. The misuse of the multiplication rule whenever they find two signs in a problem as in – 10 + 2. The students said that “minus and positive makes minus”(pointing at the negative sign in \_ 10 and an addition operation sign)
2. Multiplying the signs of numbers as in – 6 – (– 2). They said that ‘minus and minus become plus (Pointing at the negative sign of 6 and 2), thus obtaining 6 – 2 = 4.



1. Neglecting the sign of larger number and then subtract or add the rest as in

– 6 – 2 = 6 = 4. They neglected a negative sign of 6 to obtain 6 – 2.



1. Incorrect use of distributive law as in – 2 – (– 6). They took out a negative sign and placed brackets – (2 + 6) = –8.



These results indicate that, students in Brunei were doing better in multiplication and division of integers compared to addition and subtraction of positive and negative integers. An error noted when there are two negative signs in a problem on multiplication or division, is that, students only take one negative sign and ignore the other as in –7 × –3 = –21. The underlying misconception is that, the sign of one number disappears. The post-testing results led to a conclusion that the Jar Model was less confusing than the number line model and created better understanding for students compared to the rules and analogies that teachers were previously fond of using. Furthermore, the study urges teachers to use certain words for operations like, take away or add instead of using only the terms minus or plus.

## 

## 2.9 Studies Done in Tanzania

There have been a few studies done on teaching and learning mathematics in Tanzania. Wangeleja (2003) conducted a study to find out factors that inhibit the effectiveness of primary school mathematics teachers from teaching the subject. The study involved 12 teachers and 50 pupils from six primary schools in Dar es Salaam. Classroom observation techniques were used. The main factors revealed by the study were: shortage of instructional materials, incompetence on the part of some of the teachers with regards academic and pedagogical skills, and lack of adequate school management support. Specifically, the researcher noted that teachers had no lesson plans, and presented poor lessons characterized by excessive use of non-interactive teaching strategies, poor classroom management and non-use of teaching aids. Also, large class size, overcrowded classrooms and poor working and learning conditions were other militating factors.

In 2005, the Ministry responsible for education in Tanzania, MOEC issued a report (MOEC, 2005) on a study whose focus was on achievement of Standard VI pupils and their teachers in both Mathematics and reading Literacy in Tanzania. The study was part of work of the Southern Africa Consortium for Monitoring Educational Quality (SACMEQ II). The study was conducted in 1999-2000 and involved some schools selected from all eight Inspectorate Zones in Tanzania Mainland.

The findings were presented based on the Rasch model, which was used to locate levels of competency against levels of difficulty of test items to assess mastery of skills or competencies in mathematics of Standard VI pupils and their teachers. Eight levels of competencies were identified. A pupil reaching one level, would have an even chance of getting an item right in that level, but would not have chances to get an item right at higher levels. This may due to lack of mastery of skills or competencies required to get an item right at those levels. Details and percentages of pupils reaching each competence level were as follows: (Levels 7 and 8 were mainly meant for teachers).

*Level 1*: *Pre-Numeracy.*

In this level,pupils were tested to:

1. Apply single step addition or subtraction operations
2. Recognize simple shapes
3. Match numbers and pictures, and count.

Only about 2.8% of the pupils could reach this level. All the teachers reached this level.

*Level 2: Emergent numeracy*

Pupils were tested to:

1. Apply a two steps addition or subtraction operation involving carrying, checking or converting pictures to numbers
2. Estimate the length of familiar objects and recognize them
3. Recognize the common two-dimensional shapes.

Only about 22.7% of the pupils could reach this level, while all the teachers reached this level.

*Level 3: Basic numeracy*

Pupils were tested to:

1. Translate verbal information presented in a sentence into a simple graph or table, using one arithmetic operation repeated several times.
2. Translate graphical information into fractions
3. Identify place value of whole numbers up to hundreds
4. Apply simple everyday units of measurements

Only about 35% of the pupils could reach this level. All the teachers reached this level.

*Level 4:*  *Beginning numeracy*

Pupils were tested to:

1. Translate verbal or graphic information into simple arithmetic problems
2. Apply several arithmetic operations (in correct order) on whole numbers, fractions, and/or decimals.

Only about 1.5% of the pupils and 21.4% of the teachers could reach this level.

##### *Level 5: Competent numeracy*

##### Pupils were tested to:

1. Translate verbal, graphic or tabular information into arithmetic form in order to solve a given problem
2. Solve multiple-operation problem (using correct order of arithmetic operations) involving common units of measurement and/or whole and mixed numbers.
3. Convert basic units of measurement from one level of measurement to another.

Only about 2.7% of the pupils and 9.9% of the teachers could reach this level.

*Level 6: Mathematically skilled*

Pupils were tested to:

1. Solve multiple operation problems (using correct order of arithmetic operations) involving fractions, ratios and decimals.
2. Translate verbal and graphic representation of information into symbolic, algebraic and equation form in order to solve a given mathematical problem.

Only about 6.2% of the pupils and 13.2% of the teachers could reach this level.

*Level 7: Problem solving*Pupils and their teachers were tested to extract and convert information from tables, charts,visual and symbolic presentations in order to identify, and then solve mult-step problems.

Only about 1.6% of pupils and 38.8% of the teachers could reach this level.

*Level 8:*  *Abstract problem solving*

Pupils and teachers were tested to (mainly for teachers) identify the nature of an unstated mathematical problem embedded within verbal or graphic information, and then translate it into symbolic or equation form in order to solve the problem.

Only about 0.4% of the pupils and 43.9% of the teachers could reach this level.

From these findings of competence levels, one can deduce that about 25% of Standard VI pupils in the country reached the first and second levels and therefore, they were still not numerate. One can also conclude that 35% of the pupils could not go beyond the basic numeracy skills, 21.4% could not go beyond beginning numeracy skills and 9.9% could not go beyond the competent numeracy skills. The findings of this study have provided a good assessment of primary school pupils on numeracy competencies. The question remaining unanswered by the study is what difficulties the pupils were encountering when attempting question items in the different levels and how to overcome them.

## 2.10 Knowledge Gap

In Tanzania little is known about the readiness of Form I entrants to learn secondary school mathematics. As noted in this chapter, the competence of Form I entrants in numeracy skills is requisite for them to learn secondary mathematics successfully. Though learning misconceptions are the main cause of errors in solving mathematics problems, there is no evidence to suggest that; teachers know the categories of errors and underlying misconceptions that Form I entrants may have in numeric skills. This was likely to be attained and a problem removed if teachers are administering a diagnostic test to the entrants, and take remedial action. The Baseline manual recommended by the ministry of education to prepare Form I entrants for learning secondary school subjects including mathematics, is addressing mainly the subjects’ language aspects. Therefore, the manual suffers both subject content and pedagogical approach for meeting the needs of Form I entrants in numeric skills. This study attempted to bridge this gap by employing diagnostic approach to identify errors and misconception committed by Form I entrants in numeric skills, and designed a pedagogical remedial approach comprising of 12 elements to be considered by the teacher.

**2.11 Concluding Remarks**

The literature review presented in this chapter has tried to show the magnitude of the problem worldwide. The review also shows the general concern among academicians outside Tanzania. In Tanzania the learning of mathematics at primary and secondary school levels is still hampered by various problems including pedagogical approaches, and no effort has been made to address the issue of incompetence in numeracy skills amongst students at primary and secondary school levels. This study is an effort towards a full understanding of the problem in the context of Tanzania. The next chapter presents the methodology used in this study.

## CHAPTER THREE

## 3.0 RESEARCH DESIGN AND METHODOLOGY

## 3.1 Introduction

This chapter presents the research design and methodology. It begins by presenting the area, population and sample of the study, before it turns to design and approach, data collection techniques, data analysis, issues of reliability and validity and finally the ethical considerations in this study. Each of these elements is discussed in relation to the purpose and objectives presented in chapter one and the literature gaps identified in chapter two.

**3.2 Research Design and Approach**

**3.2.1 Research Design**

Various researchers (e.g. De Vaus, 2001; Kothari, 2004; Mlyuka, 2011) have put forward explanations on what is and what constitutes research design. Kothari (2004) explains the research design as a conceptual structure within which research is conducted, and that it constitutes the blueprint (design) for the collection, measurement and analysis of data. With similar view De Vaus (2001) identifies several blueprints for planning researches. These include one; case study design which attempt to conduct an in-depth study of a particular research problem rather than a statistical survey.

It is often used to narrow down a very broad research into one or a few easily researchable examples. Two; experimental design which enables the researcher to maintain a control over all factors, that may affect the results of an experiment. In this design a researcher attempts to determine or predict what may occur. Three; action research design that plans, implements, reviews and evaluates an intervention for improving practice or solving a local problem. Four; descriptive design used to obtain data about the existing status of the phenomena, and to describe ‘what exists’ with respect to variables or conditions in a situation.

Mlyuka (2011) includes issues of relevance and purpose to the explanation of research design. He contended that research design is an arrangement of conditions for collection and analysis of data in a manner that aims to combine relevance with the research purpose. It is also argued that “generally research design guides a researcher in the process of collecting, analyzing and interpreting data for achieving the objectives of the study’ (Mlyuka, 2011:14).

Taking these explanations into consideration this study has employed a descriptive design. Descriptions of errors that students make in mathematics, misconceptions and causes as well as interrogations of the students and the way mathematics teachers perceived the ability of Form I entrants aimed to achieve the purpose of the study which was to investigate the competence of Form 1 entrants in numeric skills in order to find the most effective remedial measures to take.

**3.2.2 Research Approach**

This study employed techniques of both quantitative and qualitative methods. Quantitative and qualitative approaches are the two general broad ways of data collection and analysis. According to Cohen *et* *al*. (2000) the selection and employment of a particular research method depends on the way reality is perceived in a given study. If on one hand the world is viewed as natural, hard, real and external to the individual, the methods will be from a range of options in the quantitative approach which include survey and experiments. If on the other hand the view stresses the importance of subjective experience of individuals in the creation of social world, Cohen *et al.* urge that, the approach should take on aspects from qualitative as well as quantitative methodologies.

This approach is similar to what Creswell and Clack (2011) term as ‘merging data approach’. In merging approach, the integration consists of combining the qualitative data in the form of text with the quantitative data in the form of numeric information. The integration is achieved by reporting results together in a discussion section of a study, such as reporting first the quantitative statistical results followed by themes that support or refute the quantitative results. This integration also can occur through the use of tables or figures that display both the quantitative and the qualitative results. In this study, the numeric data showing the computational work of respondents were merged with texts results on misconceptions and underlying causes by presenting them in tabular form, and as such, the discussion of the findings utilized a combination of texts and numerical data.

**3.3 Area of the Study**

Pre-tertiary education was not a union portfolio in the United Republic of Tanzania. As such this study was done in Tanzania mainland only. This part of the Republic had 25 administrative regions that were grouped into eight School Inspectorate zones. The study was conducted in nine secondary schools from Eastern Zone that comprised of Dar es Salaam, Morogoro and Coast regions. The zone was purposely chosen due to the possibility of reducing costs in terms of time, funds and accessibility and its representation of rural and urban contexts of Tanzania. While respondents from schools in Dar es Salaam may well depict the urban characteristics, those from Morogoro and Coast regions could portray the rural characteristics. This combination of schools could well depict the situation in the rest of the country.

## 3.4 Population, Sample Size and Sampling Procedure

**3.4.1 Target Population**

The target population of the study would include all Form I entrants enrolled in all secondary schools in the Eastern Zone and their mathematics teachers. However, it was not possible to visit all secondary schools in the zone. As such, a representative sample of schools, students and teachers was selected for the study. A carefully drawn sample made the accomplishment of the objectives possible, and enabled the researcher to be able to overcome factors such as the expenses and time which were likely to prevent him from gaining information from the whole population. A caution was taken to ensure that the sample was representative of the whole population in question.

### 3.4.2 Sampling Procedure

### 3.4.2.1 Selection of Sample Schools

Secondary schools in Tanzania are categorized as *Governmen*t*, Community*, *Non-Government* or *Seminaries.* According to Naum (1998) sampling enables the researcher to concentrate on a specific area that is representative of other areas. In this study purposive sampling technique was employed to obtain an unbiased sample of schools. The zone had 532 secondary schools, of which 17 were Government schools, 149 were Community schools, 164 were Non-Government schools and 33 were seminaries. Nine representative schools were selected from all four categories. The selection of eight schools was done from a list of Top-Ten in each Category-ranking and one school was selected from a list of Bottom- Ten in the Overall Zonal-ranking in the FTSEE -2007 Zonal results.

Each of the four schools categories was represented by two schools, that is the school which attained the highest average, and the one which attained the lowest average in the said examination as ranked in their respective categories. An exception was made in the category of community schools, from which three schools were selected. Selection of the third school was influenced by its position in the said examination. That is, a school which attained the lowest average in the Zonal ranking was also included in the study. Generally, among the selected schools two of them were on the list of the Overall Top-Ten schools, and one school was in the list of the Overall Bottom- Ten schools in the Zonal ranking. Other six schools were in between.

The following were the sampled schools with their subject percentage averages in brackets:

1. Government schools: *Mzumbe* (62) and *Zanaki* (25)
2. Community schools**:** *Mtibwa* (32), *Mbagala* (21) and *Mkamba*(9)
3. Non -Government schools: *Feza Boys* (78) and *St.Joseph Millenium* (53)
4. Seminary schools: *St. Mary’s Visiga* (76) and *Nur-Islamic* (20)

Four schools, Zanaki (Girls), Mzumbe (Boys), St. Mary’s Visiga (Boys) and Feza Boys were single sex schools, the rest were co-education schools. The selected schools were labelled G1SB: Government School(1) for Boys; G2SG: Government School (2) for Girls; C1SGB: Community School(1) for Boys and Girls; C2SGB: Community School (2) for Boys and Girls, C3SGB: Community School (3) for Boys and Girls, NG1SB: Non-Government School (1) for Boys; NG2SGB: Non-Government school (2) for Girls and Boys; S1SB: Seminary School (1) for Boys; S2SGB: Seminary School (2) for Girl and Boys.

**3.4.2.2 Selection of Students**

Literature on sample sizes tends to favour large samples in heterogeneous populations (Cohen *et al*., 2000; Bouma, 1996). In relation to secondary schools in Tanzania, a Form I class constitutes a group of students with different academic backgrounds, and thus differing in academic abilities. Eligible students for the study were all Form I students in the selected schools. Random sampling technique was employed. Form 1 students were requested to count numbers and those who stated even numbers were selected. To ensure equality in gender representation, Form I boys and girls in the co-education schools were requested to sit separate rooms and count numbers. All selected students sat for the MA-Test.

### 3.4.2 .3 Selection of Teachers

Purposive sampling was used to select teachers. Fifteen teachers were involved in the study. These were all teachers of mathematics in the sample schools as they had been teaching Form I classes for some years, and hence they were considered eligible to respond to the questionnaire.

## 3.4.3 Sample Size

The sample consisted of 460 respondents, who were 445 students and 15 mathematics subject teachers in selected schools. The population size was as depicted in Table 3.1.

**Table 3.1: Sample Size**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **S/N** | **Name of school** | **Number of participating students** | **Number of participating teachers** | **Total sample** |
|  | Mzumbe | 45 | 2 | 47 |
|  | Zanaki | 38 | 3 | 41 |
|  | Mtibwa | 40 | 2 | 42 |
|  | Mbagala | 52 | 2 | 54 |
|  | Mkamba | 39 | 1 | 40 |
|  | Feza Boys | 73 | 2 | 75 |
|  | St. Joseph Millenium | 50 | 1 | 51 |
|  | St. Mary’s Visiga | 45 | 1 | 46 |
|  | Nur-Islamic | 63 | 1 | 64 |
| **Total** | | **445** | **15** | **460** |

**3.5 Types and Sources of Data**

Both primary and secondary data was collected for the study. For the purpose of this study the primary data collected was specifically intended to meet the requirements of the problem under study, which was to identify the types of students’ errors in mathematical computations, underlying misconceptions and causes. Primary data collected in this study were the computational errors in numeric skills, students’ misconceptions on rules and procedures, causes of misconceptions and views of teachers on competence of Form I entrants in mathematics.

Secondary data was collected for the purpose of finding out the ability of Form I entrants in mathematics as reflected in the PSLE, as well as the ability of Form II students in the mathematics- FTSEE. These data were accessed from relevant published and unpublished documents and reports from MOEC, sampled schools and NECTA.

### 3.5.1 Data Collection Techniques

**3.5.1.1 Test**

The Mathematics Achievement Test (MA-Test) devised by the researcher was the main data collection instrument. The test (Appendix A) was purposely designed for diagnosing errors committed by students in mathematics. According to IGNOU (2012), such tests reveal learning difficulties when it is originally designed for diagnosis purpose. The test was comprised of fifty items; most of them picked from the PSLE- Mathematics papers for the examination cycles of 1998-2002. This was done to ensure clarity and preciseness of a test since the PSLE questions were set by NECTA experts and hence were regarded to be standard questions.

The items covered a range of mathematical knowledge and skills that were expected to have been covered at Primary school level. Since the main objective of the test was to analyze computational errors in numeracy skills, at least three questions were set on each skill aspect in order to capture any repetitions of errors on a single skill aspect. The respondents in Seminaries and Non-government schools were expected to be a mix of those from Kiswahili and English Media Primary schools. For this reason both languages were used to compose the test items. The respondents in the said schools were informed accordingly.

Administration of the test was done by head of Mathematics department in the sample schools. Respondents were asked to read the questions very carefully before giving their responses. They were also encouraged to ask questions on what was not clear. The speed factor was not taken into consideration. Instead, ample time was allowed for respondents to respond to as many items as possible. This was done for the purpose of ensuring that all respondents had a chance of attempting all questions in order for the researcher to find out areas which were not learnt properly for each tested skill, and hence to locate the exact source of difficulty. A marking scheme for the test was prepared so as to minimize subjectivity in marking respondents’ scripts. The test and marking scheme were scrutinized for content validity by two subject experts from NECTA.

**3.5.1.2 Questionnaire**

Questionnaire (Appendix B) was used to collect information from all mathematics teachers in the sample schools. The aim of the questionnaire was to elicit teachers’ responses about the ability of Form I entrants in numeracy skills, and the measures taken at school level to assist weak students. A questionnaire was chosen because the respondents had adequate time to give well thought out answers in their own words, and free from the influence of the interviewer. In addition, respondents who were away from their work stations could also be contacted easily. The questionnaire consisted of open and closed-ended questions.

With the help of the heads of department, it was easy to distribute the questionnaire during tea break, and to collect them before they went home. Before the questionnaires were given to teachers, the researcher met them, and explained the aim of the study; this helped to have questionnaires filled in time.

**3.5.1.3 Documentary Review**

Documentary review was another technique used in the collection of data. Documents reviewed were: Form No.9 ‘Takwimu za Shule za Msingi (TSM 9 Forms) for all Form I entrants in the Government, and Community sample schools; samples of the Form I entrance examination in mathematics; results of selected entrants into the Non-government secondary schools; and samples of Form I Mathematics orientation course programmes, that were administered by some non-government secondary schools. These helped in the following ways: Forms No. 9 were used to access the scores obtained by candidates in mathematics at the PSLE; a sample of the Entrance examination was used to review the mathematical concepts and skills tested in school-based mathematics entrance examination in non-government schools; results of the Entrance examination helped to identify the mathematical ability of respondents in non-government schools, and lastly a sample of Form I orientation programme by MOEC helped to identify the content of the mathematics orientation course programme and its adequacy in addressing the problem. It was anticipated that the content of the course programme would either provide a remediation of mathematical difficulties noted in the results of the entrance examination, or present a review of difficulty concepts in the primary school mathematics. A documentary guide used in the study appears in Appendix D.

**3.5.1.4 Focus-Group Discussion**

This study also employed the Focus-Group Discussion (FGD) technique in collection of data for triangulation or / and proving the conjectures made on causes of misconception. In order to select students for participation in FGD, one school was randomly selected from each category; i.e. government, community, non-government and seminary categories. Thereafter, four students in each school were selected to participate in the discussion. These were from each of schools G2SG, C3SGB, NG2SGB and S2SGB.  The students in each group were purposely selected including two students who got the lowest scores in the MA-Test, and another two who got average scores. Identification of the students was done using the attendance list because each script was labeled using the student’s serial number in the list. Through discussion, students were asked to give reason for the answers they gave, which in reality they provided wrong answers for short questions in the FGD guide. Their explanations led to identification/confirmation of the misconceptions they had on conceptual knowledge and application of rules and methods, as well as the underlying causes.

The technique also acted as a means of cross-validating some information collected through the questionnaire. This technique also, allowed the researcher and respondents to talk freely about the problems hindering effective teaching and learning of mathematics at primary school level. Appendix C shows the structured questions used in the FGD.

## 3.6 Data Analysis Plan

Guba and Lincolin (1994) describe data analysis as a systematic process involving working with data by organizing and breaking it into smaller manageable portions. They add that, data needs to be synthesized in order to discover what is important and what has been learnt in order to decide what to tell others. Since the study used both qualitative and quantitative data, techniques of both methods were used for data analysis. Quantitative data were summarized and presented in tables before they were analyzed by simple statistical computations of frequencies and percentages.

The scores of the Form 1 entrants in the PSLE were tabulated to indicate the range of their performance in grades. A marking scheme for the competence test was prepared and used for marking the scripts. The scores were analysed to show the percentage of those who attempted each of the questions, percentage scored in each item, and percentages of those who got wrong answers in each item. The analysis was done using the Statistical Package for the Social Sciences (SPSS). For each item, the common errors made and the associated misconceptions were noted. In the same way the true conjectures regarding the causes of the observed misconceptions were identified. Data obtained from the questionnaire was categorized and analysed qualitatively so that frequencies of responses and percentages were calculated and tabulated for easy interpretation. Content analysis techniques were used to analyse the data collected in line with the purpose of the study. Presentation of FGD data came during the discussion phase as a way of triangulating the information obtained quantitatively or ascertaining the conjectures.

### 3.7 Reliability, Internal and External Validity

**3.7.1 Reliability**

Reliability refers to how consistent a research procedure or instrument is. Reliability can be tested by finding out what were the sources of data and whether the data were collected by using proper methods (Kothari, 2004). In this study, different sources of information were used by examining evidence from the sources to build justification.

Reliability of the MA-Test used in this study was improved by placing at least two items for each skill tested; choosing Form I entrants and their mathematics teachers as the main source of information; administering a test in all sample schools almost at the same time just after the Form I students had joined the schools, and before they were taught secondary school mathematics. Also, the researcher elaborated about some items whenever they were deemed unclear to respondents. Both, the MA-Test and the questionnaire were pre-tested at one secondary school in Dar Es Salaam (not included in the study) in order to be sure that they measure what they are supposed to. Data obtained from the pre-testing exercise enabled the researcher to modify some of the instruments, as deemed appropriate. For example, some items in the teachers’ questionnaire were removed after pre-testing. The use of multiple research instruments, allowed the researcher to observe the consistency of data collected, and hence to ensure the reliability of information collected.

**3.7.2 Validity**

When the instruments used in the study enables one to get what was desired, then there is validity. Validity therefore, refers to the quality that a procedure or an instrument used in the research is accurate and meaningful. Strategies that were used to validate the instruments in this research included the use of multiple research instruments. The researcher used three types of instruments to obtain more accurate information. These instruments were MA-Test, a questionnaire and a focused-group discussion guide. MA-Test responses were clarified through the focused-group discussion, as well as some responses obtained through questionnaire. This combination of instruments contributed to record what was intended to record as well as to measure what implied to measure.

## 3.8 Ethical Dimensions

Cohen *et al*. (2000) note that, ethical issues are likely to be raised by social scientists from the kinds of problems investigated with regards the methods used to obtain data. They explain further that, each stage in the research sequence may be a potential source of ethical problems. Such ethical issues depend on the extent to which the willingness and collaboration of the participants are guaranteed, promise of confidentiality, and thorough discussion on potential consequences of their involvement in a particular study. A research clearance letter by the Vice Chancellor, The Open University of Tanzania (Appendix E) was used to introduce the researcher to the Regional and Districts authorities in the three regions (Dar es Salaam, Morogoro and Coast). Responsible officers in the said regions and districts were requested for permission to allow the researcher to collect data from their schools. Copies of the questionnaire and FGD guide prepared for research purpose were attached with the letter for showing the nature of data required from the schools. Also, the purpose of this study was made clear to them that, it was only for fulfillment of the requirement for the degree of Doctor of Philosophy (PhD) - Education at The Open University of Tanzania.

The researcher discussed with the responsible officers in the regional and districts offices, heads of schools, as well as respondents about the intention and benefits of the study. This was aimed at informing them that, as education stakeholders, they were likely to benefit from the study. For the purpose of confidentiality, all schools from which data was collected were all assigned code names, the questionnaires for all teachers and MA-Test papers for students did not ask for names and could therefore not be tracked except through the researcher. Collected responses were accessible only to the researcher, supervisors and a few individuals involved in providing advice for data analysis and discussion.

Consequently, confidentiality of responses from individual respondents was guaranteed. However, the respondents could have access to the conclusions of the findings for this study from the University after the thesis is submitted and accepted.

**3.9 Concluding Remarks**

A descriptive research design was employed in this study for which the aspects of both qualitative and quantitative methods were employed. Clustering and purposive sampling was used to select schools and teachers respectively. Random sampling technique was employed to obtain students for the study. Great efforts were made to maximize reliability and validity of the various operations employed in the process of construction and administration of research instruments. The next chapter presents the data collected, its analysis and discussion.

# **CHAPTER FOUR**

# **4.0 DATA PRESENTATION, ANALYSIS AND DISCUSSION**

## 4.1 Introduction

This chapter focuses on presentation, analysis and discussion of the data collected. The aim of data presentation and analysis is to answer the research questions and address the research objectives which specifically sought to:

1. Investigate the competence of Form I entrants in the Primary School Mathematics.
2. Investigate the common errors committed by Form I entrants in numerical calculations.
3. Identify the mathematical misconceptions leading to errors in numeric skills.
4. Conjecture on the possible causes of the errors and misconceptions.
5. Design an appropriate mathematical remedial approach for removing the noted errors and misconceptions.

The findings of the study are organized and presented in sections 4.2 to 4.7. Section 4.2 presents the competence of Form I entrants in mathematics, section 4.3 presents the difficult levels of test items, section 4.4 presents the diagnosis of errors, misconceptions and causes, section 4.5 presents the identified common errors, misconceptions and causes, section 4.6 presents the pattern emerging from the major findings, section 4.7 presents the designed format of remedial class lessons.

## 4.2 Competence of Form I Entrants in Mathematics

The first objective was to investigate the competence of Form I entrants in the primary school mathematics. The 2009 PSLE results in mathematics were used to give an overview of the performance of Form I entrants in mathematics. A mathematics achievement test was then administered to 455 respondents to investigate their competence in solving primary school mathematics; with a view of identifying errors they commit in solving the problems, underlying misconception and causes. Finally some of the respondents were involved in a focus group discussion to put in sharp focus the root causes of the misconceptions that led to incorrect answers in the test. A test of conjectures on the root causes of the observed misconceptions was also carried out. The findings were as presented in subsections 4.2.1 to 4.2.4.

**4.2.1 Performance of Form I Entrants in PSLE-Mathematics Subject**

Table 4.1 show ranges of scores attained in mathematics PSLE (2007) by the Form I entrants in the five schools (3 Community and 2 Government) as revealed from Takwimu za Shule za Msingi (TSM 9 Forms).

**Table 4.1:** **Range of Row Cores of** **Form I Entrants in PSLE-Mathematics Subject in Five Secondary Schools**

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Schools** | **Range of row scores (Out of 50)** | | | | |
| **01 - 10** | **11 - 20** | **21 - 30** | **31 - 40** | **41 - 50** |
| G1SB | 0 | 0 | 0 | 15 | 24 |
| G2SG | 1 | 3 | 19 | 12 | 3 |
| C1SGB | 0 | 5 | 18 | 14 | 1 |
| C2SGB | 1 | 10 | 21 | 4 | 0 |
| C3SGB | 0 | 0 | 13 | 27 | 0 |
| **Total** | **2** | **18** | **71** | **72** | **28** |

**Source**: Regional Education Offices for DSM, Morogoro and Coast.

Analysis of scores shown in Table 4.1 show that, almost 50% of the PSLE candidates selected to join Form I in the 5 schools were weak in mathematics. As shown in Table 4.1, Form I entrants in G1SB Secondary School had the highest scores in mathematics; and all 39 (100%) students scored grades above 30/50 or 60% in that examination. This is because the school was one of a handful of secondary schools that the Government has designated as ***‘****special schools’* for talented boys. Therefore, all it’s Form I entrants were pupils from Tanzania mainland regions with an A-grade average score at the PSLE. In contrast, the majority of pupils selected to join other schools as shown in the Table 4.1 were relatively low achievers, with some having an average score of as low as C or even D (See schools C2SGB andC2SG).

Generally, with exception of G1SB (a special school), the number of candidates who scored above 30 points out of 50 was only 61 (33.6%) out of 152. This means that, more than 66% of Form I entrants scored below 30 points and thus, they were low achievers in mathematics. These findings also suggest that, a PSLE candidate do not need to pass Mathematics in order to be selected for secondary education. Unlike the Community and Government schools, the Non- Government secondary schools and Seminaries do not use the PSLE results for recruitment of Form I entrants. Instead, these schools including the four schools in this study were conducting School prepared-entrance examinations.

Unfortunately, the results of these privately administered examinations are not made public. Results and examination scripts from seminaries were inaccessible except results from one seminary S2SG where 55(87.3%) of the Form I entrants out of 63, scored either D or F grade. A similar access problem to TSM-9 was experienced in Non- Government and seminaries because the schools admit Form I entrants from different regions in Tanzania. For example, during this study it was observed that, school S1SB had Form I entrants from 9 regions. It was also noted that, setting of the entrance examination papers, and analysis of the results at school S1SB, was for the purpose of selection only. The researcher was expecting that, such examinations were also used by the schools to shade light on areas where examinees have inadequate understanding, and hence to plan for remediation.

### 4.2.2 Teachers’ Views on Performance of Form I Entrants in Mathematics and Efforts to Help Them

When asked about the ability of Form I entrants in Mathematics (Questions 12 to 15, Appendix B) nine teachers out of the 15 who had taught Form I classes, in response to item 13, six of them viewed Form I entrants as having no required mathematical skills. Among the problems they face include lackof knowledge in basic operations and of some content. The latter include mathematical tables, decimals, fractions, area, volume, algebra and word problems. Implied by these views, a considerable proportion of Form I entrants had problems in learning mathematics.

Teachers’ views about the ability of Form I entrants, and efforts made to assist them at school level, were sought through a Likert scale items. It was generally revealed that, most of Form I entrants like mathematics but they did not have the prerequisite skills of learning the Secondary Mathematics. This problem had not been resolved since the teachers had indicated the absence of intervention in the schools. Table 4.2 shows Teachers’ views on the ability of Form I entrants in mathematics and efforts made to assist the weak ones.

**Table 4.2:** **Teachers’ Views on Ability of Form I Entrants in Mathematics**

| **No.15** | **Item** | **Rating**  **N = 15** | **Strongly agree** | **Agree** | **No opinion** | **Disagree** | **Strongly disagree** |
| --- | --- | --- | --- | --- | --- | --- | --- |
| i. | Most Form I entrants have negative attitude towards mathematics | Freq**.** | 2 | 2 | 1 | 4 | 6 |
| *%* | *13.3* | *13.3* | *6.7* | *26.7* | *40* |
| ii. | Most Form I students have poor background in mathematics | Freq. | 7 | 4 | 0 | 2 | 2 |
| % | *46.7* | *26.7* | *0* | *13.3* | *13.3* |
| iii. | Most Form I entrants have inadequate skills in arithmetic operations. | Freq. | 5 | 5 | 0 | 3 | 2 |
| % | *33.3* | *33.3* | *0* | *20* | *13.3* |
| iv. | Teachers have a habit of identifying mathematical misconceptions amongst Form I entrants. | Freq. | 1 | 2 | 1 | 5 | 6 |
| % | *6.7* | *13.3* | *6.7* | *33.3* | *40* |
| v. | There is a special programme at school to help Form I entrants who are weak in mathematics. | Freq. | 0 | 2 | 0 | 4 | 9 |
| % | *0* | *13.3* | *0* | *26.7* | *60* |
| vi. | Form I orientation courses conducted in some schools are helpful in improving their mathematical skills | Freq. | 2 | 3 | 7 | 2 | 1 |
| % | *13.3* | *20* | *46.7* | *13.3* | *6.7* |
| vii. | Drilling students with revision questions can help them to improve their performance in mathematics | Freq**.** | 5 | 4 | 1 | 3 | 2 |
| % | *33.3* | *26.7* | *6.7* | *20* | *13.3* |
| viii. | It is necessary to orient Form I entrants in arithmetic skills before introducing them to secondary mathematics. | Freq**.** | 7 | 3 | 0 | 2 | 3 |
| % | *46.7* | *20* | *0* | *13.3* | *20* |
| ix. | The Form I Mathematics textbook is suitable for beginners of secondary mathematics | Freq. | 2 | 4 | 1 | 3 | 5 |
| % | *13.3* | *26.7* | *6.7* | *20* | *33.3* |

Items 15 i-viii sought the views of teachers on the competence of Form I entrants, the extent to which schools and individual subject teachers had been taking the responsibility to identify weak students and efforts made to give assistance. The findings in Table 4.2 indicates that 66.7% of teachers refuted the statement that Form I entrants had negative attitude towards mathematics as opposed to 26.6% who agreed with the statement and 6.7% who had no opinion.

As it can be deduced from Table 4.2, teachers who view Form I entrants as having either poor background in mathematics or inadequate skills in arithmetic operations were 73.4% and 66.6% respectively. About 73.7% of teachers indicated that they had no habit of identifying the mathematical misconceptions Form I entrants may have in mathematics. Moreover, 86.7% of the teachers responded that there were no special programmes in their schools for helping weak students in mathematics. It is also indicated that 33.3% of the respondents viewed the Form I orientation courses conducted in some schools as helpful in improving students’ mathematics skills.

On the strategy used to improve students’ performance, 60% of the respondents used revision questions, while 33.3% were against the strategy, and 6.7% had no response. Those who supported the use of orientation programmes as a remedy strategy were 66.7% of whom 46.7% strongly agreed and 20% who just agreed. The teachers also gave their opinions on the suitability of a Form I Mathematics textbook. As shown in Table 4.2 about 40% (including 13.3% who strongly agreed, and 26.7% who agreed) of respondents considered the Form I textbook as suitable for beginners of secondary mathematics. Generally, the views of most teachers in Table 4.2 is an indication that Form I entrants were having learning difficulties in mathematics, and that; the teachers do not diagnose students’ learning difficulties for remediation before teaching them secondary mathematics.

## 4.3 Difficult Level of Test Items

A mathematics achievement test comprising 50 questions (Appendix A) was constructed and administered to the 445 Form I entrants in the nine selected secondary schools. The test items were written both in Kiswahili and in English, because some respondents from seminaries and Non-Government schools were from English Medium Primary Schools and those from both Government and Community secondary schools were from primary schools whose medium of instruction was Kiswahili. All test items were on topics taught at primary school level as indicated in Table 4.3. The table also shows the difficulty levels of competences tested in the tasks starting with the one having highest score of percentage failure.

To ensure reliability of the test, more than two items were constructed on each tested topic as recommended in Kheong (1982). For each question, respondents were required to show the work that led to their answers. This was done for the following three reasons. Firstly, was to underscore the importance of the process (method) of arriving at an answer to a mathematical problem. Secondly, to simplify the process of identifying the errors made by respondents in answering each question, and lastly, to facilitate the process of identifying the misconceptions (incorrectly or wrongly acquired concepts) leading to the wrong answers. For each concept and skill tested, the difficulty level was measured by the percentage number of respondents with wrong answers, compared to the total number of respondents.

As shown in Table 4.3, the highest failure rates are found in questions 47, 48, 46, 44, 45, 36 and 43. These were questions on *Units of area*, *Volume*, *Capacity*, *Length*, *Time* and *Weight*, respectively. The failure rates in these items, in their respective order, range from 83.9% to 70.7%. The next category in the order of difficulty, were questions with failure rates ranging between 50% and 70% which were on: *Percentages, Angles, Number patterns, Algebra* and *Arithmetic operations involving negative numbers*.

**Table 4.3: Percentages of Students’ Failure on Competence Tested in the Task**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **No** | **Task/Question** | **Class level taught** | **Competence**  **or skill required** | **Percentage failures** |
| 47 | Badili m2 100 kuwa sm2  *Convert 100 m2 into cm2* | IV, V, VI, VII | Conversion of units of area | 83.9 |
| 48 | Badili m3 10 kuwa sm3  *Convert 10m3 into cm3* | V, VI, VII | Conversion of units of volume | 82.9 |
| 46 | Badili m3 1000 kuwa lita  *Convert 1000m3 into litres* | V, VI, VII | Conversion of units of volume | 81.9 |
| 44 | Kadiria urefu wa mlango wa chumba cha darasa lako kwa kutumia sentimita.  *Estimate the length of your classroom door in cm.* | III, IV, V, VI | Estimation of length | 76.9 |
| 45 | Kadiria muda utakaotumia kutembea kutoka mlango wa chumba cha darasa lako hadi kwenye mlango wa ofisi ya Mkuu wa Shule yako.  *How much time will you take to walk from your classroom to the School head’s office* | III,IV,V,VI | Estimation of time | 74.7 |
| 36 | Tafuta wastani wa muda ufuatao: saa 1:40, saa 2:55, saa 8:05, na saa 3:20.  *Find the average of time durations 1:40, 2:55, 8:05 and 3:20* | III,IV,V,VI | Addition of units of time | 72.7 |
| 43 | Kadiria uzito wa mtoto mchanga katika gramu  *Estimate the weight of a newly born baby in grammes* | III,IV,V,VI | Estimation of weight | 70.7 |
| 50 | Punguza meta 40 kwa asilimia 20 (20%) *Reduce 40m by 20%* | V,VI,VII | A problem involving percentages | 69.0 |
| 39 | Tafuta thamani ya  iwapo ukubwa wa pembe katika umbo lenye pembetatu ni  na  *If these are three angles of a triangle find the value* *of x.* | V,VI,VII | To use a rule on sum of angles of a triangle in forming, and solving an equation | 65.8 |
| 9 | Tafuta jibu/*Compute*: . | IV,V,VI,VII | Subtraction of mixed numbers | 64.8 |
| 49 | Kwa kutumia rula na penseli chora kwa kukadiria pembe yenye ukubwa wa nyuzi 1350  *By using a ruler and a pencil, construct angle 1350.* | V,VI,VII | Estimating angles by drawing | 64.0 |
| 34 | Andika namba mbili zinazofuata: -2, 4, 10,…  *Write the next two numbers* | VI,VII | Identifying number pattern | 63.8 |
| 37 | Iwapo na , tafuta thamani ya .  *Evaluate* | V,VI,VII | Mixed multiplication and subtraction | 60.8 |
| 40 | Rahisisha/ *Simplify*: . | V,VI,VII | Simplifying simple algebraic expression | 59.6 |
| 33 | Tafuta kipeuo cha pili cha 0.0473  *Find the square of 0.0473* | V,VI,VII | To multiply decimal numbers | 59.3 |
| 38 | Rahisisha/ simplify:  **km**  **m**  **sm**  3 25 75  5 | VI | Multiplication of units. | 57.3 |
| 41 | Tafuta thamani ya iwapo .  *Find the value of* *x.* | V,VI,VII | Solving simple equations. | 54.6 |
| 10 | Tafuta jibu la/*Find*:  – 9 – 3 | VI,VII | Subtraction of a positive number from a negative number. | 54.1 |
| 30 | Tafuta jibu la/ *Find*:  – (– 8 ÷ 4 ) | VI,VII | Division of negative and positive numbers with a negative sign | 53.8 |
| 35 | Badili 12.25% kuwa desimali  *Change 12.25% in* *decimal form.* | V,VI,VII | Converting Percentages into decimal form | 52.9 |
| 42 | Nafikiria namba fulani. Ikizidishwa kwa tatu na kisha kuongeza 4, jibu linakuwa 1. Tafuta thamani ya namba ninayoifikiria.  *I am thinking of a number, when it is multiplied by 3 and adding 4, the answer is 1. What is the number?* | VI,VII | -Creating an equation.  -Solving a simple  Equation. | 51.4 |
| 27 | Tafuta jibu la/ *Evaluate*:  – 6 – ( + 3 ) | VI,VII | Subtraction involving positive and negative numbers | 51.4 |
| 8 | Toa/ *Subtract*:  97 – 971 | IV,V,VI | Subtraction of a big integer from the small integer. | 49.6 |
| 25 | Kokotoa jibu/*calculate*:  12 – 10 × 7 | VII | Use of BODMAS rule | 47.9 |
| 13 | Toa/*subtract*:  0.89 – (– 8.9 ) | VI | Subtraction of a negative decimal number from a positive decimal number. | 47.6 |
| 15 | Gawanya/*Divide*:  7.65 ÷ 34 | VI | Division of decimal numbers by a whole number. | 47.4 |
| 22 | Zidisha/*Multiply* :  3.5 × | V,VI | Multiplication of a decimal number with a mixed number | 46.2 |
| 23 | Zidisha/*Multiply*  2.14 × 0.029 | V,VI | Multiplication of decimal numbers | 45.2 |
| 19 | Zidisha/*Multiply* | V,VI | Multiplication of a mixed number by a fraction | 41.4 |
| 7 | Toa/*subtract* 0 – 8 | IV,V | Subtraction of a positive number from Zero | 40.2 |
| 29 | Kokotoa jibu la/*Calculate*:  – 3 – (–3) | V,VI,VII | Subtraction of two negative numbers | 40.2 |
| 31 | Badili  kuwa sehemu rahisi.  *Convert  into fraction in its lowest form.* | V,VI,VII | Converting percentages to fractions | 40.0 |
| 17 | Gawanya/*Divide* | V,VI | Division of fractions | 38.0 |
| 32 | Badili namba ya desimali 2.6 kuwa asilimia  *Change 2.6 into percentages* | V,VI | Converting decimals to percentages | 37.0 |
| 28 | Gawanya/*Divide*  (+ 78 ) ÷ (– 13 ) |  | Division of positive and negative numbers | 37.0 |
| 18 | Gawanya/*Divide*:  0.12 ÷ 0.003 | V,VI | Division of decimal numbers | 35.5 |
| 20 | Zidisha: 0.29 × 1000 | V,VI | Multiplication of a decimal number with multiples of 10 | 35.5 |
| 21 | Zidisha/ *multiply* :  58 × 347 | V,VI | Multiplication of Large integers | 31.8 |
| 24 | Kokotoa jibu la/ *Calculate*  6 – 8 ÷ 2 | VII | Application of BODMAS rule. | 31.0 |
| 16 | Gawanya/*Divide*: | V,VI | Division of mixed numbers | 30.8 |
| 14 | Gawanya/ *Divide*: 9113 ÷ 13 | V,VI | Division of integers using the long method | 30.3 |
| 11 | Toa/*Subtract*: | V,VI | Subtraction of mixed numbers | 26.6 |
| 6 | Toa/*Subtract*:  2.11 – 1.173 | V,VI | Subtraction of a number with three decimal places from one with two decimal places. | 26.3 |
| 12 | Toa/*Subtract*:  7.03 – 1.174 | V,VI | Subtraction of a number with three decimal places from one with two decimal points | 24.3 |
| 4 | Jumlisha/*Add*: | V,VI | Addition of mixed numbers | 23.1 |
| 26 | Gawanya/*Divide*: | V,VI | Division with fractions | 18.1 |
| 3 | Jumlisha/*Add* :  43.003 + 14.01 + 58.507 | V,VI | Addition of decimal numbers(two stages) | 17.1 |
| 2 | Jumlisha/*Add*:  0.021 + 9.98 | V,VI | Addition of decimal numbers(one stage) | 12.2 |
| 1 | Jumlisha/*Add*: 856 + 9134 | IV,V,VI | Addition of whole numbers | 10.9 |
| 5 | Jumlisha/*Add*:  4012 + 679 | IV,V,VI | Addition of whole numbers | 6.5 |

The third category includes questions of failure rates ranging between 25% and 50%. The questions were on a wide spectrum of competences falling into four groups:

1. Subtraction of whole numbers, decimals, integers and mixed numbers
2. Use of the BODMAS rule
3. Multiplication of decimals, mixed numbers and decimals or fractions, decimals by multiples of 10, three digits numbers,
4. Division of decimal numbers, fractions, positive and negative integers, mixed numbers, integers by long division method.

The lowest range of failure rate (below 25%) was in questions involving *addition of mixed* numbers*, division involving proper fractions, addition of decimal numbers* and *addition of whole numbers.* More analytically, the operations of addition, subtraction, multiplication and division of numbers were considered separately in Table 4.4. The purpose was to determine the levels of difficulty when computing whole numbers, integers, fractions and decimals. Consequently, four ranges of failure rates were identified. That is from: 100% - 75%, the Highest range; 74% - 50%, the High range; 49% - 25%, the Average range; and 24% - 0%, the Low range.

As shown in Table 4.4, students faced the highest difficulty on subtraction when subtracting mixed numbers as well as subtracting negative from positive decimals. The same highest difficulty was also on multiplication of numbers with three decimal places, as well as multiplying decimals of more than one decimal place. They had low difficulties on addition of numbers, and average difficulties in other several areas on subtraction, multiplication and division as shown in the table.

**Table 4.4: Students’ Failure Rates on Addition, Subtraction, Multiplication and Division of Numbers**

| **Operation** | **Questions’ areas** | **Ranges of failure rates %** | | | |
| --- | --- | --- | --- | --- | --- |
| **Highest** | **High** | **Average** | **Low** |
| Addition | 1. Whole numbers |  |  |  |  |
| ii. Improper fractions |  |  |  |  |
| iii. Mixed numbers |  |  |  |  |
| iv. Decimals |  |  |  |  |
| Subtraction | i. Mixed numbers |  |  |  |  |
| ii. Positive integer from Negative integers |  |  |  |  |
| iii. Large whole number from small one |  |  |  |  |
| iv. Negative decimal from positive decimal |  |  |  |  |
| v. A number from zero |  |  |  |  |
| vi. Negative integers |  |  |  |  |
| vii. Mixed numbers |  |  |  |  |
| viii. Small decimal from Large decimal |  |  |  |  |
| Multiplication | i. Numbers with three decimal places |  |  |  |  |
| ii. Decimal by a mixed  number |  |  |  |  |
| iii. Number with two decimal places |  |  |  |  |
| iv. Mixed number by a Proper fraction |  |  |  |  |
| v. Decimal by a multiple of 10 |  |  |  |  |
| vi. Large integers |  |  |  |  |
| Division | i. Decimals |  |  |  |  |
| ii. Proper fractions of different denominators |  |  |  |  |
| iii. Positive integer by negative integer |  |  |  |  |
| iv. Large whole numbers |  |  |  |  |
| v. Mixed numbers |  |  |  |  |
| vi. Proper fractions of same denominators |  |  |  |  |
| Mixed Operations | Use of BODMAS Rule |  |  |  |  |

## 4.4 Diagnosis of Errors, Misconceptions and Causes

This section presents the common errors found in the solutions of each question, the associated misconceptions, and their causes. The presentation is organized to address question items falling under the following 12 topics:

|  |  |
| --- | --- |
| i. Units | vii. Use of BODMAS rule |
| ii. Estimation | viii. Computation involving negative numbers |
| iii. Percentages | ix. Division of numbers |
| iv. Angles | x. Multiplication of numbers |
| v. Number patterns | xi. Subtraction of numbers |
| vi. Algebra | xii. Addition of numbers |

Under each topic, the presentation begins with the item that recorded the highest failure rate through to the one with lowest rate.

### 4.4.1 Computation of Units of Measurements

Length, area, volume, capacity and time are taught at primary school level in Standard III, IV, V, VI and VII. Items 47, 48, 46, 38 and 36 sought to find out the extent of students understanding regarding a task of conversing units of length, area, volume, capacity and time.

*Knowledge and Competences/skills tested*

1. Converting square metres (m2) into square centimeters(cm2)
2. Converting cubic metres (m3) into cubic centimeters (cm3)
3. Converting cubic metres (m3) into litres (*l*)
4. Addition/conversion of minutes and hours
5. Conversion of kilometers, metres and centimeters

*Sample questions*

1. Item 47: Convert 100m2 into cm2
2. Item 48: Convert 10m3 into cm3
3. Item 46: Convert 1000m3 into litres

iv. Item 36: Find the average of the following times: 1:40, 2:55, 8:05 and 3:20.

v. Item 38: Find km m cm

3 25 75

 5

### 4.4.1.1 Converting Units of Area, Volume and Capacity

Common errors noted in solutions for items 47, 48 and 46, the associated misconceptions and causes are shown in Table 4.5.

**Table 4.5: Student failure rates (%) by Work, Errors, Causes of Errors and Misconceptions on Converting units of Area Volume and Capacity**

| **% Failure rate** | **Student’s work** | **Errors** | | **Associated misconceptions** |
| --- | --- | --- | --- | --- |
| **Noted errors** | **Causes** |
| 83.9 | If 1m = 100cm  1m2 = 100cm2  Then if 1m2 = 100cm2  100m2 = *x*  By cross multiplication  *x* =  = 10,000cm2 | Incorrect use of the relationship  1m = 100cm to deduce that    Exclusion of units in the calculation | Squaring only the prefix, instead of squaring both the quantity and unit prefix. Hence incorrect understanding of the term ‘Square unit’ | Defined 1m2 incorrectly as  1m2 = 1× m × m  instead of the correct meaning  1m2 = (1m)2  = (1m) × (1m) |
| 82.9 | If 1m = 100cm then  1m3 = 100cm3  So if 1m3 = 100m3  10m3 =  *x*  By cross multiplication  1*x* = 10 × 100  *x* = = 1,000cm3 | Incorrect use of the relationship 1m = 100cm to deduce that  1m3 = 100m3    Excluded units in the calculation | Cubing only the unit prefix instead of both the quantity and prefix. | Defined 1m3 incorrectly as 1m3 = 1×m×m×m  instead of the correct meaning  1m3=(1m)× (1m)× (1m) |
| 81.9 | If 1m = 100cm  1,000m3 = 100,000cm3  Since 1,000cm3 = 1*l*  100,000cm3 = *x*  By cross multiplication  1,000 × cm3 = 100,000cm3 × litres  *x* =  =100 litres | Incorrect use of the relationship  1m = 100cm to deduce that  1m3= 100m3 | Cubing only the unit- prefix instead of both the prefix and quantity expressed in centimeters. | Defined 1m3 as  1m3= 100cm3  instead of  1m3 = (1m)3  = 1m × 1m × 1m =  100cm×100cm×100cm  = 1,000,000cm3 |

There were two types of errors noted in the respondents’ work of converting *square metres into square centimeters, cubic metres into cubic centimeters* and *cubic metres into litres.* The first is a conceptual error. Over 81% of respondents applied incorrectly the relationships of units. This error occurred because they did not have a good understanding of the relationship between metre and centimetres. They knew that *1m = 100cm,* but did not understand the definitions or rather the expanded forms of ‘a square metre’ and ‘a cubic metre’. These respondents had a misconception that since the relationship of one metre and centimetres was that *1m = 100cm,* then *1m2 = 100cm2*. Likewise they deduced that *1m3 = 100cm3*. Clearly, the respondents squared or cubed only the prefixes, forgetting to do the same for the numerical quantities as well. Therefore, respondents lacked correct conceptual understanding, that *1m2* = *12m2 = (1m)2= 1m × 1m*. Similarly, they did not understand that *1m3 = (1m)3 = 13m3 = 1m ×1m ×1m.*

When students understand these expanded forms, it will be easy for them to establish the correct relationship that *1m2 = 1m ×1m = 100cm ×100cm* as well as the relationship *1m3= 1m × 1m× 1m = 100cm ×100cm ×100cm.*

The second error was procedural. Respondents exclude units either in the calculations or in the final answer, as indicated in Table 4.5. An effective remediation is for the teacher to discuss with students about definitions of ‘a square metre’ and a ‘cubic metre’ as well as giving them adequate exercises to practice the conversions in discussion groups and individually.

**4.4.1.2 Computing Units of Length and Time**

Conversions of minutes into hours, as well as conversion of metres into kilometers, were tested in items 36 and 38. Standard units of time are seconds, minutes, hours, days, weeks, months and years. The day is divided into 24 hours; an hour is divided into 60 minutes and a minute into 60 seconds. For this reason, addition and subtraction of minutes and seconds does not follow the usual rules of arithmetic. This is because minutes and hours work in base 60. Table 4.6 shows errors found in solutions for items 36 and 38, the associated misconceptions and causes.

Respondents were requested to find the average of the given time durations. The correct procedure is to obtain the sum of the given four quantities of time, then divide by 4. As shown in Table 4.6, 72.7% of them obtained 15hr 20min as the sum of 2:55, 1:40, 8:05 and 3:20 instead of 16hr 00min. It is evident that respondents regarded addition of minutes to be done in the same way as for base ten numerations. As such they committed *base 10 addition error* because they had no knowledge of numeration of minutes and hours.

**Table 4.6: Student Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Computation of Units of Time, and Length**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure rate** | **Student’s work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 72.7 | 2:55  1:40  8:05  + 3:20  15:20 | addition of minutes is done in the same way as for base 10 numeration. | Unable to differentiate the hours/minutes numeration from the base ten decimal numeration. | Did not distinguish decimal number notation from hour/minute notation. |
| 57.3 | **km m cm**  3 25 75  × 5  16 125 **375**  **km m cm**  3 25 75  × 5  **16** 28 75 | Did not split  *cm* into *m*      Created a ‘carrying figure’ from the product of *m* added it tothe product of *km* | Did not know the relationship between the given units of length and hence unable to make proper conversion. | Does not know the meaning of the Prefix ‘centi’ in the  word ‘centimeter’  Does not know the meaning of the prefix ‘kilo’ in the term ‘kilometre’. Thinking that  100m = 1km |

Each of the metric units of length is related to the metre. The relationship between kilometre is that, 1000metres = 1 kilometre (Kilo means 1000) and, the relationship between a metre and centimeters is that 100cm = 1m (Cent means 100). In item 38, respondents were expected to decompose the product of 75m × 5 into kilometres and metres. As shown in Table 4.6, respondents committed an error called *decomposition of units* by leaving 375cm in the column of centimetres. Remediation of these errors on converting units of time and length can be done by familiarizing students with relationships of “an hour and minutes”, “a metre and centmetres”, and “a kilometer and metres”. This familiarization should be followed by extensive exercises on changing one unit into another.

### 4.4.2 Estimating Length/Height, Weight and Time

It is very useful to be able to estimate length, mass or weight, volume and time, because it may always not be easy to measure them. Estimation of these quantities is taught in Standard IV, V and VI. In items 43, 44 and 45, respondents were requested to estimate the length of a classroom door in centimeters, the time it takes to walk from the classroom to the head teacher’s office, and to estimate the weight of a newly born baby.

*Knowledge and Competences/Skills tested*

1. Estimating heights and lengths
2. Estimating the time spent on an activity
3. Estimating the weight of a substance

*Sample questions*

1. Item 44: Estimate the length of your classroom door in centimeters
2. Item 45: How much time will you take to walk from your classroom to the head teacher’s office?
3. Item: 43: Estimate the weight of a newly born baby

Table 4.7 presents errors noted in solutions of items 44, 45 and 43, associated misconceptions and causes.

**Table 4.7: Student Failure Rates (%) By Work, Errors, Causes of Errors and Misconceptions on Estimating Length, Time and Weight**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rate** | **Student’s work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 76.9 | **Sample answers:**  Range of answers given  i. 2cm to 7cm  ii ii. 12cm to 18cm  iii. 30cm to 38 cm  iv.100cm to 120cm    v. 1000cm | Too low  estimates    Too high estimates | They were used to:  i. instruments (scale)  for measuring  length  ii. instruments with standard lengths  like a metre rule and the ‘30 cm ruler’ | Did not know the range or width of a ‘unit centimeter’ |
| 74.7 | **Sample answers**  25sec  48sec  5min  10min  50min | Very low  estimates  Very high  estimates | They were not used to measure time duration when doing something. | Did not know the duration of a second or a minute. |
| 70.7 | **Sample answers**  1.5gm,  2gms to 10gms,  100gms to 600gms,  5kg to 9kg,  10kg to 25kg | Very low estimates  Very high estimates | Were not used to describe how much is there in a substance. | Had unrealistic ‘feeling’onhow heavier a unit kilogramme is. |

Respondents were physically sitting in the classroom looking at the classroom door. The lengths of doors in their classrooms were normal, slightly more than the height of an adult person. This means that, by knowing their physical heights, which is usually more than 1.5metres, they were expected to give estimations in the range of 150cm to 200cm. Instead, they gave either too low estimates, ranging from 2cm to 40cm, or too high estimates, like 1000cm as shown in Table 4.7.

Normally, most students in the sample schools walk from school to the nearby bus stand and from home to the school or from dormitory to the classroom. Since most of them had wrist watches, they were expected to make good estimates of time durations in terms of minutes and hours. In all sample schools, the time to walk from the classroom to the head’s office ranged between 1 and 2 minutes. Most respondents gave either too low estimates ranging from 25 to 50seconds, or too high estimates, ranging from 5 to 50 minutes.

The underlying difficulty is that, students did not have any sense of ‘a second’ or ‘a minute’ duration. This is because they were not used to plan their activities in terms of time durations of minutes and hours. It was also common for students to buy some packed domestic needs whose weights are in the range of 1 to 5 kilograms like sugar, flour and rice. The weight of a baby is within the range of 2.5 to 5 kilogrames. However, most respondents gave very low estimates ranging from 1gm to 600gms, while others gave very high estimates ranging from 10 to 25 kilogrames. The diagnosis is that, respondents had no ‘sense’ of how heavy a kilogram is. When students lack the number sense, then certainly they will have difficulties in estimating or judging quantities of magnitudes.

According to Reynolds (2008) the confidence of estimating and judging a magnitude is one of the characteristics of a good number sense. However, when one knows the size of an object, then one can estimate by comparing with another which she/he doesn’t know its size. Reynolds (2008) gives several useful hints for estimating metric units such as ‘ the height of a standard door is about 2m, the length of an adult pace is about 1m, width of a pin is about 1mm, a tea spoon hold about 5ml of a liquid, 1hectar is about two standard football pitches’.

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### 4.4.3 Computations Involving Percentages

‘Percentage’ is the concept of considering values as parts of a hundred. In this regard, the percentage value is computed by multiplying the numeric value of the ratio by 100. Problems on percentages include calculations of percentage increase, and decrease of the amount, changing decimals and fractions into percentages and vice versa. At primary school level percentages are taught in Standards V, VI and VII. In this study, respondents were required to compute four items: 50, 35, 31 and 32 on percentages.

*Knowledge and Competences/skills tested*

1. Reducing a number by a certain percentage
2. Changing percentages into decimal or fractional form

*Sample questions*

1. Item 50: Reduce 40 by 20%
2. Item 35: Change 12.25% into decimal form.
3. Item 31: Express % as a simple fraction.

iv. Item 32: Change 2.6 into percentages

Table 4.8 presents respondents’ errors, misconceptions and causes in solving items 31, 32, 35 and 50. As indicated in Table 4.8, the responses show that, to reduce a number by a certain percentage was the most difficult exercise, followed by changing percentages into decimal form. On the other hand, converting a decimal number into percentages was less difficult to most respondents. Difficulties noted were mainly due to making incorrect interpretation of the terminologies used. For example, while the respondents correctly associated the term ‘reduce’ with the operation of subtraction, they incorrectly interpreted the description ‘40 minus its 20% to mean 40 – 20% = 40 – . This wrong interpretation caused them to commit an error of *using the percentage (in fractional form) as a number to be subtracted instead of using the* ‘*percentage of a number’*.

**Table 4.8: Students Failure Rates (%) By Work, Errors, Causes of Errors and Misconceptions on Computing Percentages**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure rate** | **Students work** | **Errors noted** | **Associated misconceptions** | **Causes** |
| 69.0 | 40  = 40  39.8 | Did not compute  20% of 40 | Regarded 20% =  as the amount to be reduced from 40 | Incorrect interpretation of the mathematical language used |
| 52.9 | 12.25%  = 12.25 x 100  = 1225 | Multiplied by 100 instead of dividing by 100. | Thought that the conversion is done in the same way as for changing a decimal number into percentages. | Did not understand the rule for changing percentages into decimals and vice versa. |
| 40.0 | 100 =    % = 75% | Multiplied by  100 instead of  dividing by 100  Regarded = 75 instead of 0.75 | Regarded the term ‘Percent’ to mean 100 parts instead of th parts | Did know the definition of ‘Percent’ |
| 37.0 | 2.6 = 2  =  = 216  2.6 = | Wrote2 instead of 2  Wrote 2.6 **=**  Instead of 2  Multiplied by 100% as for the case of expressing number into percentages | Thought that .6 was another way of writing  Thought that a decimal point in 2**.**6 was separating the *numerator* and *denominator* as for the short line in.  . | Did not know that decimal points are parts of 10s  Did not know:  i. the meaning of a decimal point in the decimal number as well as the purpose of the short line in the fraction  ii. the rule for changing a decimal number into fractional form |

The misconception they had, in changing percentages into fractional form was that, the procedure of converting percentages into fraction was the same as the procedure of changing a number into percentages. For this reason, they committed an error of *confusing the rule* for changing decimal number to fraction. Furthermore, respondents had a misconception that since 75% was equal to  in fractional form; therefore  % was equal to 75 % which is an error of *confusing the* *meaning of the term ‘percent’*. This implies that the respondents had poor understanding of the rule that ‘to change a percentage into fractional form, divide by 100’ which results from the definition that ‘percentage’ means hundredth () parts. The results of the focus group interview revealed that the respondents did not know the meaning of the symbol %which is another way of writing ‘Over hundred’.

Respondents also had misconceptions on changing a decimal number into fractional form. For example, they considered a decimal point which is a separator of a whole number and a decimal part to have the same meaning as a short line in the fraction which separates a numerator from denominator. The implication is that, respondents did not know the meaning of a decimal point in the decimal number, as well as the meaning of the short line in the fraction. As observed in the focused-group discussion some respondents could not tell the meaning of 2.7 and. This means, 2.7 = 2 + 0.7 whereby 2 is the whole number part and 0.7 is the fractional part, and  means 3 out of 4 equal parts.



### 4.4.4 Computations with Angles

An angle is a figure formed by two rays with the same end point. Students are taught to estimate the size of an angle before measuring it to avoid reading from a wrong scale when using a protractor. Polygons constitutes of interior and exterior angles. Questions on computing for the size of an angle of a polygon, and estimating the size of an angle by drawing, were used to assess the acquisition of knowledge about angles, and the skill of recognizing various sizes of angles. The topic of angles is taught in Standards V, VI and VII.

*Knowledge and competences/Skills tested*

1. Computing the size of angles of a triangle
2. Estimating an angle by drawing

*Sample questions*

1. Item 39:If x –15o, 2x +300 and 150 are the interior angles of a triangle, find the value of x



1. Item 49: Draw the angle 1350 without using a protractor.

Table 4.9 shows failure rates of students by errors, misconceptions and causes in solutions for items 39 and 49.

**Table 4.9: Students Failure Rates (%) By Errors, Causes of Errors and Misconceptions on Calculation of Angles**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rate** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 65.8 | (– 150) + (2x + 300 ) + 15 = 3600  = x + 2x- 15 + 300 + 150  = 3600  = x + 150 - 15 + 300  = 3600  x + 300 = 3600  x = 3600  + 300  x = 3900 | i. Wrote 15  degrees as an  ordinary number  ii. Wrote 3600 as the sum  Transferred the term +300 (from left) across equality without changing its sign as well | i. Did not know the basic facts on angles of polygons, including a triangle.  ii. Also, they did not have adequate skills for solving equations. | Believed that:  i. the sum of interior angles of a triangle is 3600  ii) The sign of a term remained unchanged when transferred to the other side of the equation |
| 64.0 |  | Drew an acute angle or a right angle.  Drew  a rectangle  or a triangle | i. Did not have basic concepts on size of angles.  ii. Unfamiliar with measures of standard angles such as 450 and 900 whose sum is 1350 | They had unclear mental picture of angle sizes. |

Looking at the work done by respondents, most of them found the task of computing the size of an angle a very difficult one compared to estimating an angle by drawing. About 70% of the respondents failed to compute the sum of interior angles of a triangle. This implies that, they did not know basic facts about angles of polygons, including triangles.

Lack of adequate skills for solving equations was also a big problem when solving for the unknown number. The error was in transferring the term across the equality sign without changing its sign. Also, they wrote figures for degrees as ordinary numbers, for example writing 15 instead of 150 as shown in the sample of students work in Table 4.9.

Most respondents had inadequate knowledge of the size of angles and how to construct them. The question was to construct the angle 1350 without using geometrical instruments such as a protractor or a pair of compass. It was expected that, since 1350 is the sum of two standard angles (900 and 450), and since 450 is one half of 900,it was a matter of estimating 900 by drawing 900 and add a drawing of one half of it. The finding was that 64% of the respondents drew either an acute angle or a right angle. Although they were not required to use geometrical instruments, it was also noted during the focused-group discussion that, the respondents were unfamiliar with the said instruments. This is an indication that, even if they were required to draw the angle 1350 by using geometrical instruments, they wouldn’t have done it.

### 4.4.5 Number Patterns

Given the first three or four terms in the number progression, students usually are required to write the next terms in the progression. This requires a student to think mathematically in order to discover the mathematical operation(s) or a trick being used. Number patterns are taught in standards VI and VII. Respondents were required to attempt item 34 which was about number patterns. Table 4.10 shows the noted errors, misconceptions and causes in solution for item 34.

*Knowledge and competence/skill tested:*

1. Identifying number pattern

*Sample question*

Item 34**:** Write two consecutive numbers in the sequence -2, 4, 10 …

**Table 4.10: Students Failure Rates (%) by Errors, Causes of Errors and Misconceptions on Patterns of Numbers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 63.8 | –2, 4, 10 . . .  4 – (– 2) = 2  and  10 – 4 = 6 | Did 4 – (–2) = 2 Hence missed the common difference of terms in the series. | Did not know  The meaning of  –(–a) | Thought that the negative sign in 2 and the operation sign (–) indicates the same thing. Hence ignored one of them. |

Respondents were required to write the next two terms in the Arithmetic Progression -2, 4, 10 . . . They attempted to find the common difference by subtracting the first term from the second term, also subtracting the second term from the third term. Answers obtained put them in dilemma since they obtained 2 and 6 respectively thus missing the common difference. The noted error was on subtracting the first term from the second term (4 – (–2) as they obtained 2 instead of 6. This *computational error* was due to the underlying misconception that the negative sign in – 2 and the operation sign (–) in the expression indicate the same thing. Through the group discussion it was revealed that, most of respondents didn’t know the meaning and expanded forms of – (– *a*). Based on the position of numbers on the number line, the negative sign outside the brackets in – (–*a*), means ‘the opposite of – *a*,’ which is +*a*. Also, based on the rule of multiplication by opening brackets – (–*a*) = –1(– *a*) = –1 × –*a* = +*a.*



### 4.4.6 Simple Algebra

In algebra variables are used to represent unknown numbers. These are used in combination with constants and operational signs. Statements based on such variables are manipulated using rules of operations for the purpose of simplifying algebraic expressions or solving equations. At primary school level, algebra is taught in Standard V, VI and VII. In this study, respondents were given four questions: item 37, 40, 41, and 42 on simple algebra.

*Knowledge and competences/Skills tested:*

1. Doing simple substitution
2. Addition and subtraction of algebraic numbers
3. Solving simple algebraic equations
4. Solving simple algebraic word problem

*Sample questions:*

1. Item 37: If  and *n* = 3, evaluate the expression 
2. Item 40: Simplify the expression 
3. Item 41: Find the value of  if 
4. Item 42: I am thinking of a number which when it is multiplied by 3 and then adding  to the result, the answer is 1. Find the number I am thinking of.

Table 4.11 shows errors noted in solutions of the four items as well as the associated misconceptions and causes.

In item 37 respondents were required to evaluate an algebraic expression *m*2 – *mn* – 2 by substituting *m*= –2 and *n* = 3. The *substitution error* as noted from respondents work, was in substituting *m* = –2 into *mn*. They wrote either (–2)2 + (–2)3 – 2 or



–22 + (–2 × 3) –2 instead of (–2)2 + – (–2)3 – 2. The misconception was that the two minus signs in – (–2)3 make a positive sign outside brackets and hence obtained

+ (–2 × 3) instead of +2(3). When subtracting a positive number from a negative as in –2– 2, some respondents obtained 0 as the answer instead of – 4. The respondents were confusing this with the fact that, one number minus the same number, the answer is 0, (*a* – *a* = 0). With this misconception they regarded –2 – 2 as subtraction of the same numbers and hence committed an *error of confusing numbers with opposite signs.* The respondents in the Focused-group discussion also gave 0 as the answer for the two expressions *a* – *a* and – *a* – *a*. They thought that the two expressions were the same.



**Table 4.11: Students Failure Rates (%) By Errors, Causes of Errors and Misconceptions on Simple Algebra**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure rate** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 60.8 | = (–2)2 + (–2)3 – 2  = –22 + –2 × 3- 2  = –4 – 6 – 2  =–12    = –22+ (–2 × 3) –2  = (4 +(–6) – 2  = –2 – 2 = 0 | –22 = –4  –2 – 2 = 0 | Did not know the rules of addition, subtraction and multiplication of positive and negative numbers. | Thought that:  (–2)2 = –(2)2  Thought that:  if *a*  – *a* = 0 then  *a* – *a* = 0. Misused the fact that a number minus its self is zero |
| 59.6 | 3*m* – 3*n* + (8*n*)  = 3*m* 3*n* + 8n  = 3*m* + + 11*n*  3m – 3*n* + (–8*n*)  = 3*m* – (3*n* + –8*n*)  = 3*m* – –5*n*  = 3*m* +5*n* | Wrong application of the rule  –ve × -ve = +ve    Wrong placing of brackets | Confusing the rule of multiplying negative signs to the rule of adding integers of equal signs.  Does not know how to group terms. | Thought that:  since  – *a* – *a*  = +*a*  then:  –*a* + –*a* = +*a*  Thought that the grouping of like terms by using brackets has no effect on signs of the coefficients. |
| 54.6 | – 1 = 2 +*x*  += 2 – 1  6 = 1  = | shifted – 1 and + *x* across  equality sign without changing their signs | Did not know the rules of:  i. Changing the sign.  ii. Collecting like terms when solving an equation. | Thought that the sign of the number/term remains unchanged when a number/term is shifted across the equality sign |
| 51.4 | Let the number be y  Then:  *y*  3 + 4 = 1  3*y* + 4 = 1  3*y* = 4 – 1  3*y* = 3  *y* = 1  Let p be the number.  Then:  p 3 + 4 = 1  p + 7 = 1  p = 8 | i. +4 was  transferred  across equality  sign without  changing its sign.  ii. Changed the sign of +1 while not transferred across the equality sign.    Computed 3+4 first  instead of p3 | Did not know the BODMAS rule. | Thought that the term is transferred across the equality sign without changing its sign  Thought that, the computation can be done by deciding which operation to perform first. |

Moreover, respondents had difficulties in simplifying algebraic expression by adding or subtracting algebraic terms. Most of them could not obtain the simplified expression of 3*m* – 3*n* – + (–8*n*) as they obtained 3*m* + 11*n* instead of 3*m* –11*n*. The error noted here was on the step of simplifying – 3*n* + – 8*n* as it appears in

3*m* – 3*n* + –8*n*. Respondents thought that, the rule for multiplying two negative numbers – *a* × –*a* = +*a* also applies for addition of two negative numbers (–*a* + –*a* = + *a*). For this misconception, they resolved –3*n* + –8*n* as +11*n*.



Another error was observed in collecting the terms with *n* as *a* variable in the expression 3*m* – 3*n* + (–8*n*). Respondents wrote 3*m* – (3*n* + –8*n*) instead of

3*m* + (–3*n* + –8*n*), as such they committed *an error* *of making incorrectly placing of brackets*. The cause of this error as obtained during the discussion with selected respondents is that, the respondents were not aware of the sign of a term. In the given expression the terms to be collected together were –3*n* and –8*n*.

Solving simple algebraic expression was another difficult area to respondents. Common error noted was the ‘*shift-sign change’ error* for which the shifting of terms across the equality sign was done without changing the sign. It was noted during the discussion that, respondents had no clear understanding of the rule of changing the sign as well as the rule of collecting like terms when solving an equation.

Respondents were also required to solve a simple word problem leading to the formation of an algebraic equation. Most of them were not able to formulate an equation from the given words, and the majority of those who formulated the correct equation failed to solve it. Errors noted include an *incorrect use of the BODMAS* rule and an error in *change of sign* when shifting terms across equality sign. It was observed that, respondents either shifted a term with no change of the sign or changed the sign of a term while the term is not transferred.

### 4.4.7 Use of BODMAS Rule

The BODMAS rule and its use is taught at Primary School level in Standard VII. BODMAS means Brackets Of Division, Multiplication, Addition and Subtraction. The rule is used to simplify a mathematical expression involving more than one of the arithmetic operations (+, –, × and ÷) by starting with opening brackets of division, followed by multiplication, addition and lastly subtraction. In this study, respondents attempted three questions: items 25, 25 and 30, on the use of the rule.

*Knowledge and Competence/ Skill tested:*

i. Applying BODMAS rule in computations

*Sample questions:*

1. Item 30: Simplify – (–8 4)
2. Item 25: Compute 12− 10  7

iii. Item 24: Simplify 6 – 8 2

Errors and misconceptions noted in solutions for items 24, 25 and 30, as well as the causes are shown in Table 4.12.

**Table 4.12: Students Failure rates (%) by Work, Errors, Causes of Errors and Misconceptions on Using BODMAS Rule**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 53.8 | − (− 8 ÷ 4 )  = − 8 ÷ 4  = − 2  − (− 8 ÷ 4 )  = + ( 8 ÷ 4 )  = 8 ÷ 4  = 2  − (− 8 ÷ 4 )  = +8 ÷ − 4  = − 2 | Neglected the negative sign outside the brackets  Multiplied the two negative signs in  − (−8, to obtain + as for the case of distributive law of multiplication  Opened brackets as in the case of distributive law of multiplication | Did not understand the two negative signs multiplied together. | Thought that the negative sign outside the brackets have the same meaning as the one inside.  Thought that division is associated under distributive property like multiplication. |
| 47.9 | 12 − 10  7  = 2  7  = 14 | Performed 12 – 10 first instead of 10 × 7 | Did not know BODMAS rule. | Thought that the computation can be done by starting with any operation. |
| 31.0 | 6 − 8 ÷ 2  = –2 ÷ 2  = –1 | Performed 6 – 8 first instead of 8 ÷ 2 |

The respondents’ work in Table 4.12 shows an error of *performing operations in the order as appearing in the expression*. Respondents got wrong answers because they had a misconception that, the simplification of the expression is done in the order in which the operations appear in the expression that is from left to the right. On the question 12 – 10 × 7, they computed 12 – 10 first instead of 10 × 7, while for 6 8 ÷ 2 they did 6 – 8 first instead of 8 ÷ 2. These errors and associated misconceptions were due to inadequate understanding of the BODMAS rule, and how to use it.



**4.4.8 Subtraction of Integers**

At primary school level, Pupils are taught to add, subtract, multiply and divide integers from Standard I to VI. Usually, many difficulties are experienced in subtraction involving negative integers or both positive and negative integers. Items 10, 27, 28, 29 sought to test respondents’ understanding of subtraction of integers.

*Knowledge and competences/Skills tested:*

1. Subtracting a positive integer from a negative one
2. Subtracting a negative integer from a negative one
3. Subtracting a negative number from a positive one

*Sample questions:*

Compute the following:

1. Item 10: –9 – 3
2. Item 27: –6 – (+3)
3. Item 29: –3 – (–3)



1. Item 28: 0.89 – (–8.9)

Errors and misconceptions committed by respondents in solving items 10, 27, 28 and 29 as well as causes of the misconceptions are given in Table 4.13.

**Table 4.13: Students Failure Rates (%) by Work, Errors, Cause of Errors and Misconceptions on Subtraction of Integers**

| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| --- | --- | --- | --- | --- |
| **Noted errors** | **Causes** |
| 54.1 | –9 – 3 = –6  –9 – 3 = +6 | Obtained – 6as theanswerinstead of  –12    Gave +6 as the answer instead of –12 | Did not know the different ways of writing *a – b*,  *–a – b* and – (+*a*)  They were also confusing the rules of adding two consecutive positive and negative integers. | Thought that, the two negative signs in‘ – 9 –3’ are multiplied to give  + 3 hence –9 + 3 = –6  Thought that the two consecutive negative signs in ‘–9 –3’ are multiplied to give a positive sign as for the case of  **–**a × –a = +a |
| 51.4 | − 6 − (+ 3 )  = 6 + 3 | i. − 6 − = +6  ii. − (+ 3 ) = + 3 | Did not know the:  i. rules of opening  brackets  ii. meaning of  − (+ a ),  – (–a) | Thought that the two minus signs in ‘ – 6 – ‘ are multiplied to give + 6 |
| 47.6 | –8.9  –0.89  –8.19 | Interchanged the terms from  0.89 – (–8.9) to  –8.9 – 0.89 as for the case of addition. | Did not know the meaning of – (– *a*) | Thought that it was not possible to subtract –8.9 from 0.89 |
| 40.2 | − 3 – (–3)  = −3 + 3  = −6 | −3 + 3 = −6 | Did not know the commutative law of addition. | Thought that  the negative and positive signs in  ‘– 3 + 3’ are multiplied to give a negative sign as for the case of  –a × +a = −a |

In this study, respondents were required to answer three questions. In the first question, they were required to compute –9 – 3. This means to subtract 3 which is a positive integer from –9 which is a negative integer. Most respondents did an *error of multiplying any two negative signs* in the expression to obtain a positive number. As it was observed in the findings, respondents obtained either –9 – 3 = –6 or –9 – 3 = +6. Both –6 and +6 were incorrect answers. As revealed from the discussion with selected respondents, such errors were due to a misconception that, the two negative signs in –9 – 3 are to be multiplied to make a positive number, as is the case in multiplying negative numbers. For this reason, it is clear that, respondents had no understanding of the rules on multiplication of positive and negative numbers. They were also not aware of other ways of expressing – *a* – *a*. That is –*a* – *a* = – *a* + – *a* = (–*a*) + (–*a*) = –2*a*. The underlying misconception was the misapplication of the rules of multiplication in addition operation. For this misconception, they multiplied the two consecutive negative signs in ‘– *a* – *a*’ to obtain +*a*.



The second question was to compute – 6 – (+3). This question is similar to the first one except that the terms are written in a different way. Two errors were committed by respondents in answering the question. One *error was to multiply any two negative signs* in the expression as previously noted, and the other error was *ignoring the minus sign* in the expression –6 – (+3). The respondents in the Focused-group discussion did not know the meaning of – (+*a*) or – (– *a*). Respondents would have obtained the correct answer if they knew that the negative sign in – (+*a*) means the opposite of +*a* (which is – a) and the negative sign in – (– *a*) means the opposite of –*a* (Which is +*a*) and hence writing correctly

– 6 – (+3) = –6 + –3 = –9.

The third question was to compute –3 – (–3). This means to subtract a negative integer (–3) from a negative integer (–3). Most of the respondents correctly wrote



–3 – (–3) = –3 + 3. From discussion groups the respondents said that they had crammed that ‘negative and negative is positive’ means that, the product of two negative signs is a positive sign. However, they did not know that – (– 3) means the opposite of – 3, which is +3. Also, they did not know that the expression – (–3) is the same as –1(–3) = –1 × –3. An error of *multiplying any positive and negative signs* was also noted in computing –3 + 3 as they wrote –3 + 3= – 6. Again, respondents had the misconception that, the positive and negative signs in the expression –3 + 3 multiply to give a negative number, as in the case of multiplying positive and negative numbers. This finding shows that, respondents had no understanding of the rules of addition; including the rule which states that ‘the sum of opposite numbers is 0’.



### 4.4.9 Division of Numbers

Division is like the reverse process of multiplication or repeated subtraction. Students at primary school level are taught to divide whole numbers, decimals and fractions in Standard V and VI. There rules or methods for dividing whole numbers, decimals and fractions. In this study, items 14, 15, 17, 18, 26 and 28 were given to respondents, to test their understanding in using the rules of division.

*Knowledge and competences/Skills tested:*

1. Dividing decimal numbers
2. Dividing proper fractions
3. Dividing mixed numbers
4. Dividing integers by long method.

*Sample questions***:**

Divide:

1. Item 15: 7.65  34
2. Item 18: 0.12 0.003
3. Item 14: 9113 ÷ 13
4. Item 28: (+78 ) ÷ (– 13)



1. Item 17:   
2. Item 16: 
3. Item 26: 

Errors and misconceptions noted in the solutions of items 15 and 18 as well as causes of the misconceptions are shown in Table 4.14.

**Table 4.14: Students Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Dividing Decimal Numbers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 47.4 | 2. 3  34 7.65  - 6.4  1.25  - 1.02  0.23 | Ignored decimal point in the dividend | Faced difficulties to recognize how many times does 34 go into 7.65 | Had no clear understanding of the rules for dividing decimal numbers. |
| 35.5 | =  = 4 | Changed the value of the quotient by multiplying it by  (=) instead of multiplying the quotient by  (1). | Had no clear understanding of equivalent fractions and the rule for removing decimal points in the quotient. | Thought that ‘removing decimal points’ separately from numerator and denominator has no effect on the value of the quotient. |

As shown in Table 4.14, respondents were required to attempt two questions (items 15 and 18) on division of decimal numbers. In item 15, the divisor was a whole number while in item 18, both dividend and divisor were decimal numbers. Most respondents (47.4%) failed to divide a decimal number by a whole number as compared to 35.5% of them who could not divide a decimal number by another decimal number. The common error noted in the working for item 15 was omission of a decimal point in the dividend for the convenience of making the division much easier. The correct way of making the division easier was to remove the decimal point from the dividend by multiplying both the divisor and the dividend by 100, or simply multiplying the expression by  (=1). Since this was not done, it appears that, respondents had little understanding of the rules for dividing decimal numbers. They also lacked logical thinking-tricks, since they could have identified the number of times the divisor goes into the dividend so as to determine the correctness of the answer obtained.



In item 18, respondents expressed the given expression in the quotient form and attempted to remove the decimal points from the numerator and denominator. The common error noted was to multiply the quotient by  (= 1/10) and thus completely changed the value of the given quotient. As such they had no understanding that; removal of decimal points is done by multiplying with a quotient factor of multiples of 10, which simplifies to 1. Therefore, respondents lacked both the knowledge of equivalent fractions, and clear understanding of the rule for removing decimal points.

Respondents were also examined on division of integers in items 14 and 28. Table 4.15 shows students’ failure rates, errors they committed, associated causes and misconceptions in solving the two items.

**Table 4.15: Students Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Division of Integers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 37.0 | (+78) ÷ (−13)  =  = 6 | Neglected the positive and negative signs in the computation | Did not know the rules for dividing positive and negative numbers. | Thought that the positive and negative signs are not involved when performing division as it applies for multiplication. |
| 30.3 | -91  13  -13  00 | Skipped a zero between 7 and 1 in the quotient. The zero was the ‘number of times 1 goes into 13. | Did not have clear understanding of using the long division method. | Thought that, it was unrealistic to write 0 as the number of times 1 goes into 13. |

In item 28, respondents were required to divide a positive integer by a negative integer, while in item 14 they were required to divide positive integers. As shown in Table 4.15, the proportion of respondents who failed item 14, was greater than the proportion of failures in item 28. The common *error* noted on division of positive and negative numbers was *omission of the positive and negative* *signs in the computation*. Respondents in the Focused-group discussions from schools C3SGB and NG2SGB appeared to know only the products of two negative numbers, two positive numbers as well as products of the numbers with opposite signs. However, some of them were confusing the result of: dividing two negative numbers, two positive numbers, negative and positive numbers. As such, they conceptualized wrongly the rules governing division of positive and negative numbers by confusing them with those for multiplication. Beyond expectation of the researcher, some respondents involved in the discussion groups could not tell the result of dividing two positive numbers.

In item 14, most respondents opted to use the long division method in dividing a four digits’ integer by an integer with two digits. As shown in Table 4.15, a proportion of 30.3% of the respondents failed to obtain the correct answer due to errors in the working steps. The common error noted was ‘*to skip the zero’* in the quotient when doing the step of finding the number of times ‘13 goes into 1’. The answer is 0 times. Respondents in the discussion groups had a misconception that, there can be no ‘zero times’. This observation was similar to the previously noted misconception on subtraction of numbers, that subtraction of a non-zero number from zero was impossible. By having this misconception, respondents brought down two digits at a time in the algorithm to obtain 13, for which ‘13 goes once in 13’ as the working shows in Table 4.15. Therefore it is imperative to note that, such respondents had no adequate understanding of dividing numbers using the long method. Table 4.16 shows errors, misconceptions and causes in solutions of three questions (item 16, 17 and 26), which sought to test students’ understanding of division of fractions.

**Table 4.16: Students Failure Rates (%) By Work, Errors, Causes of Errors and Misconceptions on Dividing Fractions**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated**  **misconceptions** |
| **Noted errors** | **Causes** |
| 38.0 | =  =  = | Used incorrect rule: | Did not understand the logic of the rule: | Thought that, the rule for dividing two fractions was: ‘multiply reciprocals of both *Dividend* and *Divisor*’. |
| 30.8 | 4  ÷ 2  =  =  =  = | Used incorrect rule: | Did not understand the logic of the rule: | Thought that, the rule for dividing fractions was: to multiply divisor by the reciprocal of dividend. |
| 18.1 | =  = 12    =  = | Used incorrect rule:      Used incorrect rule:  = | Did not understand the rule for dividing fractions. | Thought that, the rule for dividing two fractions was: ‘multiply reciprocals of both *Dividend*  and *Divis*or  Thought that *numerators* and *denominators* are divided separately as for the case of fractions. |

Generally, as shown in the table, the failure rate on these items ranged from 38.0% to 18.1%. Failure to obtain correct answers was due to the incorrect use of the rule for dividing fractions. While the correct rule was to multiply the dividend by the reciprocal of the divisor that is:, some respondents either multiplied the reciprocals of dividend and divisor, or multiplied the dividend by reciprocal of the divisor, that is ×. It was also noted that, there were respondents (18.1%) who divided the numerators and denominators separately, as shown in Table 4.16. As it was revealed during the discussion with selected respondents in school C2SGB, respondents committed such errors because they could not distinguish the rules governing division of fractions, from the rules of multiplication. Furthermore, it was also evident that, some respondents did not understand the logic of inverting the divisor in the rule of dividing two fractions.

### 4.4.10 Multiplication of Numbers

Multiplication can be considered as repeated addition. However, multiplication tables simplify the process of multiplication. This means that, regardless of multiplying whole numbers, fractions, or decimals, a good understanding of multiplication tables is required. Students at primary school level are taught multiplication in Standard IV, V and VI. In this study, six questions: items 19, 20, 21, 22, 23, 33 were given to respondents to test their understanding on applying rules of multiplication.

*Knowledge and Competences/Skills tested:*

1. Multiplying decimal numbers
2. Multiplying a decimal number by a mixed number
3. Multiplying a mixed number by a fraction
4. Multiplying a decimal number by multiples of 10.

vi.Multiplying large integers by long multiplication method.

*Sample questions:*

i.Item 33:Find the square of 0.0473

ii. Item 23: Multiply: 2.14  0.029

1. Item 22: Compute 3.5 1

iv. Item 19: Compute 10

1. Item 20: Multiply: 0.29 × 1000
2. Item 21: Multiply: 58 × 347

Multiplication of two decimal numbers which are both less than 1, was more difficult to perform than, multiplication involving a decimal number less than 1 and another number greater than 1, as shown in Table 4.17. In the former multiplication, respondents committed two common errors. The first error was to *skip the last two steps* of multiplying the multiplicand by zeros of multiplier. Secondly, was an oversight of *forgetting to add a carrying figure* in the last step of the answer. Selected respondents in the discussion groups said that it was not necessary to perform the last stages of multiplying 0.0475 by 0’s because the product will be 0 which adds nothing to the final product. For this error they had to put a decimal point at seven decimal places instead of eight places.

**Table 4.17: Students Failure Rates (%) By Work, Errors, Causes of Errors and Misconceptions in Multiplication of Decimals**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **%Failure rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 59.3 | 0.0473  × 0.0473  01419  03311  01892  .0**1**23729 | Skipped the last steps of multiplying the multiplicand (0.0473) by 0’s of the multiplier.  Did not add  ‘a carrying’ figure in the column. | Unfamiliar with multiplication of numbers less than 1. | Carelessness mistakes in following the long algorithm of multiplication. |
| 45.2  45.2 | 2.140  × 0.029  19260  4280  0000  + 0000  00062**.**060  2.14  0.029  = (2.14 x 1000)  ( 0.029  1000)  = 2140  29  = 2140  29  = 62060 | Placed a decimalpoint by counting decimal places from the wrong direction.    Used incorrectly the rule of removing decimal places when multiplying by multiples of 10. | Did not understand  Well the rule of placing a decimal point in the end product.  Did not have clear understanding of the rule for removing decimal places in an expression. | Thought that the number of decimal places in the product is counted from left to right.  Thought that, removal of decimal places has no effects on the value of an expression. |

Two more common errors were noted in multiplication of decimal numbers. One was the *use* *of incorrect order of counting decimal places* in the product. Respondents counted from right to left, instead of counting from left to right. Another error was in *removing the* *decimal points* from the multiplicand and multiplier by multiplying with multiples of 10 without finally dividing the product by the same multiplication factor. This changed the value of the expression as shown in Table 4.17. The misconceptions behind the two errors are mainly due to lack of clear understanding of the rule of multiplication by removing decimal points as well as confusion on which direction to follow, when counting decimal places in the product.

Errors and misconceptions committed by respondents on multiplying fractions were as shown in Table 4.18.

Two types of errors were committed by respondents in multiplying fractions. One of errors was to apply the rule of addition by finding the LCM of the denominators, and hence, changed the operation sign from multiplication to addition. Here, the respondents’ misconception was to confuse the rule of adding fractions with the rule of multiplying them. Moreover, some of respondents in the discussion groups could not tell whether the expression  ×  was the same as or not. It was therefore clear that the respondents had little knowledge and practice of the rules for addition and multiplication of fractions. Also, they did not know different ways of expressing the product of two fractions such as × =  .



**Table 4.18: Students Failure Rates (% ) by Work, Errors, Causes of Errors and Misconceptions on Multiplication of Fractions**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failures**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 41.4 | 10  =  =  10        = | Added the two fractions instead of multiplying them.    Reversed the rule of changing a mixed number into fractional form by performing 3×10, divide by 4, instead of doing 4×10, divide by 4    Inverted the multiplier as in the case of division. | Confusing the rule of adding fractions with the rule of multiplying them.    Did not know the meaning of 103/4 Also, they had inadequate understanding of the rule for changing a mixed number into fractional form.  Confusing the rule for dividing fractions with the rule for multiplying them. | Thought that LCM is applicable when multiplying fractions.  ----- ---------  Thought that the rule for changing a mixed number to fractional form was to ‘multiply integer part by numerator of fraction part, then divide by denominator of fraction part’  Thought that the rule for dividing fractions applies in multiplication as well. |

Another common error was to invert the multiplier as in the case of dividing fractions. The results from selected respondents in the focused-group discussion show that, they were confusing the rule of dividing fractions with the rule for multiplying them. This confusion led them to apply the two different rules interchangeably. Furthermore, some respondents used an incorrect procedure of changing a mixed number into fractional form since they multiplied the integer part by the numerator of the fraction part instead of multiplying it by the denominator, as shown in Table 4.19.

**Table 4.19: Students Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Multiplication of Decimals and Mixed Numbers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **%**  **Failure**  **rates** | **Student’s work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 46.2 | 3.5× 1  = ×  = ×  = | Wrote  as a fractional form of 3.5    Inverted the multiplier as in the case of dividing fractions. | They didn’t know the meaning of 3.5  They did not understand the logic of inverting the multiplier. | Thought that the digit 3 in 3.5 stands for numerator and 5 stand for denominator and a decimal point was a separator.  Thought that the rule for multiplying fractions was the same as the rule for dividing them. |

Table 4.19 shows that, 46.2% of respondents failed to obtain the correct answer on the question which required them to multiply a decimal number by a mixed number. The difficulty faced by the respondents was on changing a decimal number into fractional form. The error committed on this area was to *regard a decimal point as a separator of the numerator and denominator* and hence they wrote  as a fractional form of 3.5. The misconception behind this error as revealed from discussion groups was a thinking that since a short line in the fractional number separates the numerator and denominator therefore a dot (decimal point) in 3.5 had the same role as a short line. Another error noted in the respondents’ work was an *error of inverting the fractions*. In this case the multiplier-fraction was inverted as in division of fractions. Their misconception was that the rule of inverting the multiplier was also applicable in multiplication of fractions as well.

Multiplication of whole numbers was tested in items 21. Table 4.20 shows errors, misconceptions and causes in multiplication of large whole numbers.

**Table 4.20: Students Failure Rates (%) by Work, Errors, Causes Of Errors And Misconceptions on Multiplication of Whole Numbers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **%Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 31.8 | 347  × 58  277***4***  + 1735  20124  347  × 58  2776  + 1***6***35  1***8***126 | Obtained wrong product of 8 and 7  Forgotten to add carrying figures | i. Carelessness mistakes in multiplication and addition algorithms.  ii. They had no habit of checking the correctness of the end product. | Confusing the multiplication of tables 7 and 8 |

As shown in Table 4.20, the proportion of respondents who failed to perform correct multiplication of the two whole numbers was 31.8%. Using the long method of multiplication, respondents obtained incorrect products in the algorithm of multiplying the digits of top number by digits of the bottom number as shown in the table. Two dominant errors are noted in the respondents work. First error is to obtain *incorrect products* on multiplying the top number by digits of the bottom number, and the second error is *to skip a ‘carrying’ figure* when adding the products. These errors were either due to lack of understanding of mathematical tables, or carelessness mistakes in doing computations.

Respondents also attempted item 20, which demanded them to multiply a decimal number, by the number which is a multiple of 10. Table 4.21 shows committed errors and misconceptions in finding the solution.

**Table 4.21: Students Failure Rates (%) By Work, Errors, Causes of Errors and Misconceptions on Multiplication of Decimals and Powers of 10**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **%**  **Failures**  **rates** | **Student’s work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 35.5 | 0.29 ×1000  29000  = 29.000  0.29 × 1000  = 0.00029 | Moved a decimal point three places from the right to left instead of moving it from left to right.    Placed three 0’s to the left of .29 | Had no clear understanding of the rules for multiplying integers and decimal numbers by multiples of 10. | They were not sure which direction to move a decimal point.  Confusing the rule of multiplying integers by multiples of 10 with the rule of multiplying decimals by multiples of 10. |

Most respondents applied the rule of removing a decimal point one place to the right for each zero in the power of 10. This rule was much easier to apply than the long multiplication method. However, 35.5% of respondents failed to obtain the correct answer for 0.29 × 1000. The common errors featured in the respondents’ work were first; to move a decimal point in the wrong direction and second, to move a decimal point two places instead of three, since 1000 has three zeros. The selected respondents in the discussion groups had the idea of moving a decimal point either to the right or to the left, but did not know that to move a decimal point one place to the right means to multiply by 10 and likewise, to move one place to the left means to divide by 10. For this reason, respondents didn’t know that to move a decimal point two places to the left or to the right means to divide or multiply a decimal number by 100, respectively.

### 4.4.11 Subtraction of Numbers

We subtract whole numbers, fractions and decimal numbers. Subtraction of two numbers is considered as an operation of finding their difference. Just as in addition, subtracting of large whole numbers can be performed by setting numbers vertically, so that, the place value columns are formed. Subtraction of fractions requires the use of common denominators. It is also requires one to align decimal points when subtracting decimal numbers. Items 6, 7, 8, 9, 11, and 12, sought to test respondents’ understanding of subtraction of numbers.

*Knowledge and Competences/skills tested*

1. Subtraction of mixed numbers
2. Subtracting a large integer from a small one.
3. Subtracting decimal numbers of different numbers of decimal places
4. Subtracting decimals involving ‘borrowing’ more than once.

*Attempted questions:*

1. Item 9: Find 5 – 6



1. Item 11:Find
2. Item 8: Find 97 – 971



1. Item 7: Find 0 – 8
2. Item 6: Find 2.11 – 1.173
3. Item 12:Find 7.03 – 1.174

Subtraction of mixed numbers whose fractional parts had common denominator was much easier to compute, than subtraction of mixed numbers whose fractional parts had different denominators. As shown in Table 4.22, there were about 64.8% of respondents who failed to answer correctly the question on subtraction of two mixed numbers whose fractional parts had different denominators. Those who were unable to subtract mixed numbers with fractions of the same denominator were 26.6% of all respondents. The common errors noted include the *use of wrong LCM*, the *use of incorrect procedure* to change a mixed number into fractional form as well as incorrect use of the rule for multiplying two negative signs.

**Table 4.22: Students’ Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Subtraction of Mixed Numbers**

| **% Failure rate** | **Students work** | **Errors** | | **Associated misconceptions** |
| --- | --- | --- | --- | --- |
| **Noted errors** | **Causes** |
| 64.8 | 5 – 6 = – 1  = –1  5 – 6  = – 1  =  =  = | Used 3 as LCM instead of 9    Ignored the negative sign in the simplification  of a numerator. | Did not understand:  i.the term  ‘Common Multiple’ as well as  Ii ii. the rule for changing a mixed number into fractional form.  Ii iii. the rule for subtracting a large integer from a small one. | Thought that since 3 is smaller than 9 then, 3 is the LCM. That is, making comparison of words ‘Lowest’ and ‘Small’  Thought that,  the successive negative signs in –14– 6  1 multiply to make +14 |
| 26.6 | = 16  = 16  = 19    =  =  =    =  =  = | Used 1 as LCM instead of 3    Used 9 as LCM  instead of 3    For each mixed number, the numerator was subtracted from the whole number. | Did not know the meaning of LCM and how to find it.  Did not know how to change a mixed number into fractional form. | Thought that the LCM of the two fractions is obtained by multiplying their denominators.  Thought that, a mixed number is changed into fractional form by subtracting the numerator from the whole number. |

Respondents created a positive sign in the expression –14 – 6 as in the case of – (– 14) whose expansion form is; –1 × –4. Respondents therefore, were not able to differentiate ‘a negative sign’ from ‘a minus sign’ in the expression –14 – 6. Also, they did not know that – 14 + –6 was another way of writing – 14 – 6. In finding the ‘Least Common Multiple’ of denominators, respondents had associated the term ‘Least’ with the word ‘smallest’. This misconception led to a wrong choice of LCM. For this reason, most of them used 3 as the LCM of 3 and 9 to simplify  – 



Furthermore, some respondents committed an *error of using an incorrect procedure* for changing a mixed number to a fractional form. They subtracted the numerator of a fractional part from the integer instead of changing the integer part into fractional form with the same denominator, and added the two fractions. The cause of these identified misconceptions is lack of knowledge of the concept of negative numbers, different ways of rewriting terms involving negative integers and misunderstanding of the rule for multiplying negative numbers. Also, respondents in the discussion groups had no understanding that; a mixed number consists of a whole number and a fraction. Therefore, they didn’t know the meaning of a mixed number.

**Table 4.23: Students Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Subtraction of Integers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **%Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 49.6 | 971  − 97  874    97( )  − 971  1 | Interchanged the terms, from 97–971 to 971– 97  Incorrect alignment of digits and  -Wrong application of the vertical rule of subtraction. | Had no idea of subtracting a large integer from a small one.  Did not understand the rule for subtraction using the vertical method. | Thought that, subtraction is  commutative  Thought that the vertical method is applicable only, when subtracting large integer from a small one. |
| 40.2 | 0 − 8 = 0  0 − 8 = 8 | Obtained 0 as the answer instead of  –8    Obtained 8 as the answer instead of  –8 | Thought that it was not possible to subtract 8 from nothing (0) | Did not know the concept of subtracting a large integer from a small one. |

Table 4.23 shows that, 49.6% of respondents did not score the question on subtracting a big integer from a small one. Most of them used the vertical method of addition for which they committed an error of making incorrect alignment of digits. It was also noted that, some respondents interchanged the order of numbers in the question by computing 971– 97 instead of computing 97 \_ 971. Selected respondents from schools C3SGB and S2SGB said that it was easier to subtract a small integer from a larger one. Their argument was a confusion that, the commutative property applying in addition and multiplication does also apply in subtraction of numbers.



However, even some of those who interchanged the numbers committed the same error. The misconception among the respondents was that the commutative law is applicable to subtraction as it is to addition. Table 4.23 also indicates that 40.2% of respondents could not give the answer for 0.8. For most of them thought that, it was not possible to subtract eight units from nothing as it was raised in the discussion with selected respondents. Some respondents ignored 0 in the given expression and wrote –8 as the answer, while others simply wrote 8 as the answer. As such, one can rule out that the respondents did not know the concept of subtracting a large integer from the small one. Also, it appears that, the respondents were not aware of the use of the number line in performing addition and subtraction of integers.



According to Sadi (2007), there is a large gap between students understanding of the natural numbers and their understanding of decimal numbers. It is pointed out that many students face difficulties with concepts of decimal numbers than with any other number concept. Table 4.24 gives a sample work of respondents on subtraction of decimals.

**Table 4.24: Students’ Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions in Subtraction of Decimals Numbers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **%Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 26.3 | 2**.**110  − 1**.**173  937  1.173  − 2.11  -1.063 | The answer has no decimal point  Computed  1.173–2.11  instead of  2.11–1.173. As such they interchanged the numbers. | Did not know the significance of ‘a zero’ on the left of a decimal point.  Had no clear understanding of the rules for subtracting decimal numbers. | Thought that a zero in 0.937 can be ignored as in the case of a zero in 937.0  i. Thought that **.**173 is larger than **.**11 therefore a large number can not be subtracted from a small.  ii. Thought that decimal parts **.**11 and **.**173 were independent of integer parts |
| 24.3 | 7.030  −1.174  6.**9**56  7.03( )  −1.174  5.86**4** | Obtained 6.956 as the answer instead of 5.856    No subtraction was done in the thousandth place value. The digit (4) was simply brought down to the thousandth place value in the answer. | The idea of ‘borrowing’ is not clearly understood.  ------------  They had no idea of placing zero(s) to fill gap(s) in a number with fewer digits. | Thought that ‘borrowing’ is done only once  Thought that if a digit is missing in the vertical alignment, then no subtraction can be done in the respective column. |

Numerous errors were noted on subtraction of decimal numbers as cited in the respondents’ work in Table 4.24. First, in the question to find 2.11– 1.173, the digits were correctly vertically aligned with a replacement of a zero in the thousandth place value, to fill the gap. However, there was an *error of* *omitting a decimal point* as it was observed in the final answer (see Table 4.24).The misconception was that a zero to the left of a decimal point can be ignored as in the case of zeros to the right after the last non-zero digit. That is, they omitted 0 in 0.937 (to obtain 937) as in 937.0 = 937. This finding is similar to that of Sadi (2007). Students face a dilemma as to whether to write or to omit a zero(s) since they were told that zero in the decimal number at the end of last non-zero digit has no value and therefore can be omitted leaving the value of a decimal number unchanged.



Secondly, there was an *error of interchanging the positions of decimal numbers* for the convenience of doing easy subtraction in the decimal parts. That is, subtraction of decimal parts: .173 – .110 was much easier to perform than .110– .173. With this in mind, they perceived wrongly that, the two decimal parts were independent of respective integer parts, and that, it was impossible to subtract .173 from .11 since .11 with two digits was considered to be smaller than .173 with three digits. According to Sadi (2007), common and wrong method used by the students when adding decimals, is *to add the numbers before the decimal points,* and then, *to add numbers after the decimal points* separately. Second, was the *digit gap-filing error* as it was observed in computation of:



7.03( )

1.174



5.864

Having the gap not filled by ‘a zero’, the respondents assumed there can be no digit in that space. During the group discussion, one respondent commented that ‘nothing minus 4 equals 4’. Definitely, respondents did not know the technique of filling gaps when subtracting decimals with different number of digits. It is also obvious that, they did not know the value of zero or zeros appearing in the decimal part after non-zero digit(s). Fourthly, was an error of *borrowing from zero* for which respondents failed to borrow from zero as shown in the following subtraction:

7.030

– 1.174



6.956

In this working, the respondent did correctly the borrowing in the first two steps, but failed to borrow from zero (tenth digit). The dilemma of borrowing from zero was also raised by Sadi (2007) who views the problem of borrowing from zero as the most common problem that students have in the process of subtraction.

### 4.4.12 Addition of Numbers

Addition is the operation of putting together two or more numbers, to make another number which has the equivalent value. We add whole numbers, fractions and decimal numbers. When adding large whole numbers, it is often more convenient to set out the numbers vertically, in such a way that, corresponding digits form place values columns. Knowledge of common denominators is required in adding fractions. Also, we need to align decimal points, when performing addition of decimal numbers. Addition of numbers is taught at primary school level in Standard I to VI. Respondents were given items 1, 2, 3, 4 and 5 to test their understanding in performing addition of numbers.

*Knowledge and competences/Skills tested*

1. Adding mixed numbers
2. Adding more than two decimal numbers of different decimal places.

iii. Adding decimal numbers involving ‘to carry a figure’ more than once.

iv. Adding integer numbers with different number of digits.

v. Adding integers

*Attempted questions*

1. Item 4: Add
2. Item 2 Add 0.021 + 9.98
3. Item 3: Add 43.003 + 14.01+ 58.507
4. Item 1: Add 856 + 9134
5. Item 5: Add 4012 + 679

Ability of respondents in adding fractions was tested in item 4. A fraction is a part of a whole thing. We can think of fractional parts as equal shares of equally-sized parts of a whole or unit. More precisely, a fraction can be defined as any numbers ‘*a*’ and ‘*b*’ expressed in form of  such that *b* ≠ 0, *a* is called numerator, and *b* is called denominator. In order to add or subtract fractions, we need to make sure they are in the same family, meaning that, the denominators must be the same. This requires adjustment of the fractions so that they have a common denominator. Findings of the competence of students in adding fractions are shown in Table 4.25.



**Table 4.25: Students Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Addition of Fractions**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
|  | = 12 | Obtained  as a sum of | i. Did not understand the rule for adding fractions  ii. Did not understand the rule for converting a mixed number into fractional form. | Thought that the sum of fractions is obtained by adding their numerators and denominators separately. i.e.  = |
|  | = | Applied LCM before changing numbers into fractional form. | Did not understand:  i. the rule for changing a mixed number into fractional form  ii. the conditions  for using LCM. | Thought that LCM is used even in the mix of whole numbers and fractions. |
|  | =  + | Added together the denominator, whole number and numerator when changing the two mixed numbers into fractional form. | Did not understand  the rule for changing a mixed number into fractional form. | Thought that the rule for changing a into form of  was  ((a + b) c instead of  ((a + b) c |

As shown in Table 4.25, the respondents committed some errors in adding mixed numbers. Common errors noted included the *error of adding numerators and denominators* *separately* when finding the sum of proper fractions, and *incorrect use of the rule for* *changing a mixed number* *into fractional form*. The sum of  was easily obtained by adding integers separately, and adding fractional parts separately as well. Mistakenly, they obtained as the sum of  + by doing.



The misconception emanates from the rule of multiplying fractions, where numerators and denominators are multiplied separately. As Siegler *et al.* (2010) argue, the respondents did not know that denominators define the size of the fractional part and numerators represent the number of parts. The rule for changing a mixed number in the form of a** where ‘a’ is the integer part and  is a proper fraction is: to convert the integer part to a fraction with the same denominator as the fractional portion, andthenadding the two fractions. In this case the integer part ‘a’ is converted to a ×. The sum of the two fractions becomes:

+   =   and not +   =   as it was misconceived by the respondents (See Table 4.25). The cause for these misconceptions is inadequate knowledge of mixed numbers, little understanding of applying the rule for adding fractions, as well as a misunderstanding of the rule for converting a mixed number into fractional form. More findings indicated that, respondents added a whole number to the denominator of a fractional part when attempting to change a mixed number into fractional form. This observation is in line with that of Siegler *et al*. (ibid) who observe that, students mistakenly add a whole number part to a denominator of a fractional part.



The algorithm of adding decimals is similar to that of adding whole numbers. It is therefore important to align the decimal points in respect to their place values and to keep correct order to avoid adding tenths to hundredth or hundredth to thousandth. Normally, zeros are used to fill in the blanks when decimal numbers to be added have different numbers of digits. The skill of adding decimals was also tested in this study. Table 4.26 shows the findings.

**Table 4.26: Students’ Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions on Addition of Decimal Numbers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 17.1 | 43.003  14. 01  + 58.507  115.501 | Incorrect alignment of decimal digits | Had no clear understanding of the rule for adding decimal numbers by using vertical method | Thought that digits of a decimal part are arranged from the right as in the case of adding whole numbers |
| 12.2 | 0.021  + 9. 98  9.119 | Incorrect alignment of decimal digits |

Given an expression in horizontal form, almost all respondents wrote the expression in vertical form by aligning the digits irrespective of their corresponding place values, as shown in Table 4.26. Common errors noted included the use of *incorrect vertical* *alignment of digits* and *skipping ‘a carrying figure’*. For example, 0.021 + 9.98 was incorrectly vertically arranged as; 0.021

+9. 98

The misconception is that, the digits of the decimal part are to be vertically aligned from right to the left as in the case of whole numbers. Despite of this misconception, some respondents in the focus groups discussion thought that, a zero can be placed to fill the created gap in the tenth place value so as to have 9.098 instead of 9.98. It was further noted that, some respondents regarded the zero for the hundredth place value in 0.021 as a repetition of the zero in the ones place value. Due to this false understanding they skipped its place value when aligning the digits of the two numbers. This signifies the lack of knowledge on place values of digits forming a decimal part and/ or a little understanding of using the method of vertical addition.

In items 1 and 5 respondents were tested to perform addition of whole numbers. Table 4.27 shows common errors and misconceptions committed by respondents in this area.

**Table 4.27: Students’ Failure Rates (%) by Work, Errors, Causes of Errors and Misconceptions in Addition of Whole Numbers**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **% Failure**  **rates** | **Students work** | **Errors** | | **Associated misconceptions** |
| **Noted errors** | **Causes** |
| 1 0.9 | 856  +9134  17694 | Incorrect vertical alignment of digits | They were confusing the rule of aligning digits of large integers with the rule of aligning decimal parts. | Thought that digits of numbers are aligned vertically from the right as for the case of aligning digits in the decimal part. |
| 6.5 | 4012  + 679  10802 | Incorrect vertical alignment of digits |

As shown in Table 4.27, the common error noted in addition of the integers was incorrect use of the vertical method of addition. Digits of numbers were not correctly aligned in their respective place values**.** It was observed that,the digits of the bottom number were arranged vertically from the right as in the case of adding decimal numbers and vice versa. For example:

322 + 32 when correctly arranged vertically it becomes 322

+ ( ) 32

While 3.22 + 3.2 when correctly arranged vertically it becomes 3.22

+ 3.2( )

The correct way was to arrange the digits of the bottom number vertically below the top number, by aligning the digits from right to left. It is clear that, respondents had no understanding of place values, and did not know how to use the vertical method of addition. The misconception is on the dilemma to choose the correct direction of alignment of digits as it is also noted by Said (2007).

**4.5 Summary and Discussion of Major Findings**

Subsections 4.5.1 to 4.5.7 present the summary and discussion of major findings.

### 

### 4.5.1 Types of Errors and Misconceptions on Computations of Units of Measurements

Respondents committed two types of errors: the *unit conversion error* and *quantity* *estimation error*. The respondents had prior knowledge that 1m = 100cm. However, they made mistakes in converting m2 into cm2, m3 into cm3 and cm3 into litres. First, was to make an incorrect reasoning that if 1m = 100cm then 1m2 = 100cm2 and 1m3 = 1000cm3. Respondents appeared to have the misconception that, only the prefix is squared or cubed in the relationship 1m = 100cm. This was a conceptual error for they didn’t know the definition of a ‘unit square metre’ (1m2) and a ‘unit cubic metre’ (1m3). Students need to be introduced to the basic knowledge on area, volume and the definitions of area and volume, together with class work to be discussed in groups with the help of a teacher.

It is very useful to be able to estimate lengths, masses, time, volume and angles, because it may not always be easy to measure them (See Busbridge and Womack, 1991). In due respect, the estimation of common quantities was another tested area. In this study more than 70% of the respondents gave unrealistic estimates of length, weight, time and angle, hence generated an error type we call *quantity estimate error*. While some respondents estimated the height of their classroom to lie between 2cm and 38cm, others gave the height as 1000cm.

On the other hand, some respondents gave estimates of a baby’s weight as ranging from 1.5gm to 10gm, while other estimates ranged from 9 kg to 25 kg. All these were shocking estimates from students. Sometimes you can estimate simply by comparing one object with known size to another whose size is unknown. Results from focus group discussions had shown that, the students had no useful guide for making estimations. They also had no mental feeling/ picture of a unit kilogramme, metre or minute.

*Base-numeration error was* also manifested in the addition of minutes and hours. Respondents thought that addition of minutes and hours is done in the same way as addition of base ten numerations. Furthermore, the respondents committed another error known as *unit’s decomposition error.* The interviewed respondents were knowledgeable on the conversion from metres to kilometers, and centimeters to metres. But some of them could not split 375 cm into ‘3m and 75cm’.

They also committed *a carrying* *figure error* by creating 1km from 128m. The misconceptions underlying the committed errors in units of measurements were due to the teaching and learning approach of concepts of units of measurements, which do not consider the use of instruments inside and outside the classroom. It was revealed through the discussion with selected respondents, that respondents were not familiar with instruments for measuring length, time, weight and volume. As an example, most of interviewees could not use a ruler to measure the length of a textbook, and it was also difficult for them to measure its weight using a spring balance. This was mainly because they could not read the scales of these two instruments. With this observation the students did not know the span of a unit centimeter as well as a feeling of 500gms.

Generally, the underlying causes of these errors were due to both lack of basic concepts of units of length, unfamiliarity with the instruments, and carelessness on decomposing the units.

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### 4.5.2 Types of Errors and Misconceptions on Simple Algebra

Algebra is an important topic in the school mathematics curriculum. Its application is found in all current branches of mathematics, and science in general (Seng, 2010). Algebraic expressions contain variables, constants and operational signs. Students’ understanding of this content and the meaning helps to reduce algebraic expressions correctly (ibid.).

This study has revealed that students have difficulties in solving problems in algebra. Their difficulties are revealed in the errors and misconceptions they commit in solving questions in algebra. Three types of errors: *Incorrect placing of* *brackets, incorrect bracket opening* and *misapplying the rule* *–ve × –ve = +ve* were spotted in respondents’ work sheets on simplification of algebraic expressions. When collecting the like terms in 3*m* – 3*n* + (–8*n*), respondents placed brackets by detaching the sign from the term –3n, writing 3*m* – (3*n* + – 8*n*) instead of 3*m* +



(–3*n* + – 8*n*). Usually, the effect of sign detachment is easily detected in terms with negative coefficients rather than in terms with positive coefficients. The respondents in the focused group discussion appeared to be unaware of the coefficients of terms and the effect of grouping terms without considering their signs.

The dilemma of using a negative sign, and a minus sign was also observed in simplifying –2 –2 and – 22. The respondents wrote –2 – 2 = 0 and –22 = 4. One respondent pointed out during the discussion that, ‘a number minus itself is equal to 0’. On the bases of this rule they incorrectly perceived –2 –2 as –2 minus –2. Apparently, this means the number minus itself. The interviewed respondents thought that 22 is equal to 2 × –2. They considered 22 to be the same as (2)2, instead of 2)2. This misconception led them to the incorrect simplification that –22 = –2 ×–2 = +4



### 4.5.3 Types of Errors and Misconceptions in Computations of Fractions and Decimals

Writing numbers in decimals is another way of writing numbers in fractions. Findings of this study have revealed numerous errors in computations involving fractions and decimals. An error noted when changing a decimal number into fractional form was to *replace a decimal point by a short line,* thinking that both the decimal point and the short line had the same purpose of separating the numerator from denominator. The decimal point in 3.5 separates the whole number from the decimal part, while the short horizontal line in  separates the numerator from the denominator. These two notations have different purposes as follows: While the decimal point marks the place where numbers change to less than 1, the short line in division is a separator of the numerator and denominator. On this type of errors, Suffolk (2004) contends that, students do not know the meaning of a decimal number like 3.5 as well as the meaning of a fraction like. The same observation was noted during the focus group discussions as it appeared difficult for the majority of respondents to give the meaning of 9.2 that is 9.2 = 9 + 0.2. The error can be corrected if students have clear understanding that the numbers to the left of the decimal point are whole numbers, while the numbers to the right are decimal fractions, and that a number with 1 decimal place is a fraction with denominator 10. Hence to realize that 3.5 means 3 +.



Procedural error of doing incorrect multiplication was noted when changing a mixed number into fractional form. A mixed number is a number that has a combination of a whole number and a fractional part, with a numerator smaller than its denominator (Suffolk, 2004). As it was revealed from the focus group discussion, the respondents did not know that mixed numbers can also be written using the addition sign between the whole number part and the decimal part, that is 5 = 5 + . The correct procedure of changing a mixed number into fraction form was to multiply the whole number part by the denominator of the fraction part, and then add the numerator of the fraction part. Incorrectly, respondents multiplied the whole number by the numerator of the fraction part, a finding similar to the study by Siegler *et al*. (2010). It is therefore important to observe that, clear understanding of the relation between mixed numbers and improper fractions, and how to translate each other, is crucial in helping students to work with fractions.



Two more common errors on fractions were found in this study. These were *adding fractions using wrong LCM, and incorrect use of the rule of dividing fractions.*  The respondents failed to find the least common denominator when adding or subtracting fractions with unlike denominators. It was expected that after listing down some multiples of each denominator, the next step was to identify the common multiples from the list, thereafter to pick the smallest one. An error committed in this area, which was also observed by Siegler *et al*. (2010) is to *choose the ‘smallest multiple’* instead of choosing the *‘smallest common* *multiple’*.

After failing to convert fractions to common equivalent denominators, students just use the larger of the two denominators in adding or subtracting fractions with unlike denominators. Siegler *et al*. (2010) give two causes of this problem as follows: The first cause was that, students do not understand that different denominators reflect different-sized unit fractions, and secondly, students do not understand that adding and subtracting fractions requires a common denominator. They recommended the use of the number line, and other visual representations that show equivalent fractions as very helpful to students.

The rule for dividing a fraction is that ‘to divide by a fraction one turns it upside down and multiply’ (Suffolk, 2004). This rule requires one to turn upside down only the multiplier leaving the multiplicand unturned. The findings of this study show that respondents committed an error of *turning upside down both fractions of the multiplicand and multiplier*. This error arises because students learnt the rule without understanding the reason for it. This comment concurs with what Siegler *et al*. (2010) pointed out that students often misapply the invert-and-multiply procedure for dividing by a fraction because they lack conceptual understanding of the procedure.

In the same way, the authors point out two more errors which were also noted in this study. These were *‘not* *inverting either fraction’* and ‘*inverting the multiplicand fraction instead of the multiplier’*. Students who did not know the rule of ‘invert and multiply’ believed that the numerators of the fractions are to be multiplied separately, likewise, the denominators. This was evident in this study during the focus group discussions. Busbridge and Womack (1991) argue that, division of fractions and decimals depends on the concept of equivalence for which the division is written as ‘double decker fractions’ like ÷  = / .



Multiplication and division of decimal numbers were other areas where students committed errors. After doing the multiplication algorithm of two decimals, respondents were uncertain about how to put a decimal point in the product. They committed an error of *counting decimal places from the left side of the product* instead of counting from the right side. Also, when removing decimal places from the multiplicand and multiplier the respondents *multiplied only by multiples of 10, forgetting to divide* *by the corresponding power of 10* so as to leave the value of an expression unchanged. A similar error was also committed in division of two decimal numbers, where the top and bottom numbers were treated separately and therefore *multiplied the top and bottom numbers by different multiples of 10*.

The misconception was to think that the numerator and denominator existed separately and hence can independently be treated when removing decimal places. With these results it appears that, the respondents knew that the digits move one place to the left when multiplied by 10, two places when multiplied by 100, and so on and hence removing decimal places. However, they did not know the concept of equivalence of numbers that, a number remains unchanged when multiplied by 1. This means that, a number can be changed to an equivalent fraction by multiplying its numerator and denominator by the same number.

On the other hand, it was found out from the focus group discussion, that students did not know how to write a whole number in fractional form, for example 5 = . Hence it was difficult for them to understand that 2.14 × 0.029 =.  Causes of these errors can be explained partly due to poor understanding of the rules that govern multiplication and division of decimals (Graeber and Campbell, 1993).

Multiplication of decimals follows four rules: first, multiply the numbers just as if they are whole numbers; second, line up the numbers on the right, do not align the decimal points; thirdly, starting on the right, multiply each digit in the top number by each digit in the bottom number, just as with whole numbers; fourth, add the products, and lastly, place the decimal point in the answer by starting from the right and moving the point the number of places equal to the sum of decimal places in the numbers multiplied. Important points to consider when dividing decimal numbers are: first, the divisor must contain no decimals; second, move the decimal point in the divisor to the right until the divisor is a whole number; and thirdly, move the decimal point on the dividend by the same number of places as the divisor. Findings of this study partly concur with the finding of Sadi (2007) that, some students face a challenge of applying mechanically the properties of natural numbers to other real numbers.

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### 4.5.4 Types of Errors and Misconceptions on Computations of Percentages

Percentages are another way of writing numbers. The symbol % means ‘Per-hundred’. Basic types of problems for solving percentages include: finding a percentage of a quantity, expressing a part as a percentage of a whole and calculating percentage increase or decrease. In this study, respondents were required to reduce 40 by 20%. They misinterpreted the words “reduce 40 by 20%” by writing 40 –. As it can be seen, this misinterpretation led to an *error of using the given percentage in fractional form as a number to be* *subtracted, instead of using the percentage of the number.* Failure of understanding English language was a big factor behind this error. The Mathematical Development, Secondary survey report by the Assessment of Performance Unit ([APU], 1980) cited in Sadi (2007) shows the similar finding, as reporting that only about half of 15 years old pupils could work out ‘ 50 percentage of 250’.



Two more common errors on percentages as revealed in this study are an *error of confusing the meaning of the term percent* and *error of confusing the rule of changing a decimal number to fraction form.* The cause of such errors is certainly a failure to compute the percentage value. The percentage value is computed by multiplying the numeric value of the ratio by 100. For example to find the percentage of 3 balls out of 4 balls, first compute the fraction  = 0.75, and then multiply by 100 to obtain 75%. Also, the percentage value can be found by first multiplying 3 by 100 to give 300, and then dividing this result by 4 to give 75%.



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### 4.5.5 Types of Errors and Misconceptions on Computations of Whole Numbers

This study has revealed several errors in addition, subtraction, multiplication and division of whole numbers. Addition of whole numbers by the vertical method is done by lining up numbers along their digits and summing up the columns starting from the right and moving left. If the sum of a column is greater than 10, the tens digit is ‘carried’ over to the next column while the units digit of the column sum represents the unit digits for the answer.

Similarly, subtraction of whole numbers is done by lining up numbers so that their digits match and the number to be subtracted is on the bottom. Starting from the right, the bottom digit in each column is subtracted from the digit above it. If the digit being subtracted is larger, the next digit of the top number is lowered by one while the current digit has 10 added to it. This is called “borrowing”. In this study three types of errors were noted in addition and subtraction of whole numbers. These were: *error of making incorrect alignment of digits in addition and subtraction*, *error of skipping carrying figures* *in addition,* and *error of interchanging the arrangement of numbers in subtraction for easier computation*. These types of errors are also cited in Siegler *et al*. (2010), Suffolk (2004) and Resnick (1982). Students’ misconceptions on use of laws which govern addition and subtraction of whole numbers were the main cause of such errors. Teachers should use clear examples to show where the laws are applicable and where not applicable.

Errors identified in this study on multiplication of whole numbers were *error of obtaining incorrect products,* and *error of skipping carrying figures*. When using the vertical method of multiplication, a student needs to understand the multiplication algorithm. For example, when multiplying two digit numbers, say 43 × 12, first arrange the numbers vertically. Then, multiply the top number by the bottom digit on the right. This is the one’s digit, and in this case the one’s digit is 2. The product is 86; write 86 so that 6 is in the one’s column under the 2. Next multiply by the tens’ digit, which is 1. The product is 43 tens; write the 43 so that the 3 is in the tens’ column under the 1. Add the two products as vertically arranged. The said two errors were committed in the stages of multiplying the top number by the bottom digits, and adding the products. Carelessness and little understanding of multiplication tables are causes of observed errors. Suffolk (2004) on the same type of errors urges students to practice a 10 by 10 multiplication table, as well as mini-multiplication tables for completion. Furthermore is for teachers to teach products for which no ‘carrying’ is required, and with small numbers before students are introduced to multiply numbers which require ‘carrying’.

In this study, two common errors were noted in division of whole numbers by the long division method. First, was an *error of borrowing twice* when supposed to do it once. Second, was an *error of dropping two digits at once in the algorithm of the long division method.* The underlying causes and misconceptions are due to little understanding of the method. As Miller (2013) recommends, before a student is taught long division, he/she should first know multiplication tables, the basic division, that is based on multiplication tables; and basic division with remainder. As for multiplication, it is also recommended to check the answer by multiplying back or by estimation.

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### 4.5.6 Types of Errors and Misconceptions in Using the BODMAS Rule

The common error committed by the respondents when solving an expression with mixed arithmetic operations was an *error of executing the operations in the order they appear in the expression from left to right, irrespective of the BODMAS rule.*  Respondents had no clear understanding of the BODMAS rule. An error of evaluating the operations in the wrong order as explained by Sadi (2007), is one of the common errors made by the students when attempting to evaluate a mathematical problem involving two or more different types of arithmetic operations.

The rule requires that, when **different types of** arithmetic operations are involved in an expression, and in the absence of any brackets, operations are done from left to right. This rule states that operations enclosed by brackets are evaluated first. The use of brackets is probably to draw attention to correct order of operations. Otherwise, expressions with several operations may be given without brackets, and are then performed in the order, starting with division if it is there, followed by multiplication, addition and subtraction. Example: 20 – 6 ÷ 2 × 4 = 20 – 3 × 4 = 20 – 3 × 4 = 8. According to the rule, division is performed first, that is 6 ÷ 2 = 3; then follows multiplication, that is 3 × 4 = 12, and lastly subtraction, 20 – 12 = 8.

### 4.5.7 Types of Errors and Misconceptions in Subtraction of Integers

Understanding arithmetic operations with positive and negative integers is very important to students since the topic provides a base for progression to other topics. In this study, respondents were tested on their understanding of subtraction of integers.

The findings reveal three types of errors in subtraction of positive and negative integers. These are error *of multiplying two nearby negative signs, error of multiplying two adjacent positive and negative signs,* and *error of ignoring the minus sign in an expression.* Many rules on computations with integers are a problem to students to know them and get their meaning (Hart, 1981). Findings of this study have revealed the confusion in using the multiplication rules in addition and subtraction operations. Slight misapplication of the rules such as the rule ‘two negatives make a positive’ was noted as a common mistake among respondents. Ways to solve the problem as discussed by Khalid and Baderudin (2011), Miller (2012) and Suffolk (2004) are: to teach the meaning of rules on operations with integers rather than teaching on practicing rules only; to use patterns for which students are asked to observe the answers so as to discover the rules; use of several strategies in teaching subtraction of integers rather than relying only on the use of a number line.

**4.6 The Pattern Emerging from the Major Findings**

The pattern emerging from the major findings is that, with the exception of a ‘special school’ for talented boys, majority of Form I entrants were low achievers in mathematics. The responses from the subject teachers also revealed that most Form I entrants have inadequate skills for learning secondary mathematics, and that, there are no efforts made at the school level to identify the mathematical disabilities facing the students in order to plan for intervention. The results of the competence test revealed the difficulty levels of mathematical concepts and skills in basic arithmetic operations.

The most difficult concepts and skills for which 83.9% to 70.7% of respondents failed were the conversion of units of measurements and estimation of quantities. Next was a group of concepts for which the failure rates ranged between 70% and 50%. The concepts and skills in this category were on percentages, angles, number patterns, simple algebra and subtraction involving negative numbers. The third group of difficult concepts and skills was in the use of the BODMAS rule and arithmetic computations involving integers, fractions, mixed numbers and decimals. The failure rates in this category ranged between 50% and 25%.

The findings also indicate that 24.3% to 6.5% of respondents could not answer correctly questions on simple additions of numbers. The general overview of the findings shows that, of the four basic arithmetic operations, addition operation presents respondents with least difficulties as reflected by the proportion of respondents ranging from 25% to 6.5%, who failed to add whole numbers, integers, fractions and decimals, while the failures rates on subtraction, multiplication and division of the same type of numbers ranged from 50% to 25%.

Various errors and misconceptions were noted. Respondents committed conceptual as well as procedural errors. Carelessness errors were minimal. According to Seng (2010), procedural errors are errors caused by using incorrect mathematical operations, while conceptual errors are errors caused by selecting inappropriate model and application of rules. The observed misconceptions are due to the lack of conceptual knowledge, incorrect use of mathematical rules, and false generalization. Table 4.28 presents a summary of the identified difficult content, categories of errors and associated misconceptions.

As shown in Table 4.28, the study has specified 20 areas of difficulties in the content of primary school mathematics in Tanzania. From these areas, 33 types of errors are identified with 27 underlying misconceptions.

**Table 4.28:** **Specified Difficult Content and Corresponding** **Categories of Common Errors and Misconceptions**

| **Problem areas** | **Specific difficult areas** | **Common errors noted** | **Category**  **of errors** | **Misconceptions/**  **causes** |
| --- | --- | --- | --- | --- |
| Units | Conversion of units | 1. Incorrect use of relationships. 2. Exclusion of units in computation | Conceptual/  Procedural | Thinking that, the exponent  applies only to the prefix |
| Estimations | Making unrealistic estimates of length, weight and time. | Conceptual | Lacking a ‘feel’ or ‘sense’ of unit quantities or standard units of measurements |
| Drawing unrealistic angle estimates | Conceptual |  |
| Addition of hours and minutes length | Added units of time in the same way as for base 10 numeration | Conceptual | Does not differentiate base 10 numeration from the minutes and hours numeration |
| Multiplication of km, m and cm by the factor. | Writing a product which is not matching with a prefix | Conceptual | Does not understand the meaning of given prefix |
| Algebra | Simplifying algebraic expression by collecting like terms | 1. Incorrect placing of brackets when collecting like terms 2. Incorrect opening of brackets | Procedural/Conceptual | Thinking that brackets have no effect on the signs of the grouped terms. |
| Making direct substitution in an algebraic expression | Ignoring the coefficient of the variable or term | Procedural/Conceptual | Thinking that a variable is independent of its coefficient |
| Solving word problem | Making incorrect interpretation of mathematical language/descriptions | Conceptual | Confusing the meaning of mathematical terminologies |
| Solving algebraic equations. | Transferring a term across the equality sign without changing its sign | Conceptual/  Procedural | Does not understand the essence of the rule of changing the sign |
| Solving for angles | 1. Use of incorrect sum of interior angles of a triangle. 2. Writing angle sizes as ordinary numbers | Conceptual/  Carelessness | Thinking that the sum of interior angles of a triangle is 3600. |
| Decimals and Fractions | Decimal and fractional notations | Incorrect changing of decimals into fractional form and vice versa. | Conceptual | Considering a decimal point to have the same meaning as a short line in a fraction. |
| Changing mixed numbers into fractional form | Changing a  into form of  by doing  (b + a) ÷ c | Procedural | Thinking that the expanded form of  a  was + |
| Multiplying decimal numbers | Placing a decimal point in the product by counting decimal places from left instead of counting from the right. | Procedural | Confusing the direction of counting decimal places in the product. |
| Evaluating the quotient of decimal numbers by the method of removing decimal points. | Using two different factors (multiples of 10) to multiply the numerator and denominator. | Conceptual | Thinking that the decimals of numerator and denominator are independent numbers |
| Adding fractions with different denominators | Choosing the smallest multiple of denominators to be the LCM | Conceptual | Associating the term ‘Lowest’ as it appears in the abbreviation LCM to mean just a ‘small’ multiple. |
| Dividing fractions | 1. Multiplying reciprocals of both multiplicand and multiplied 2. Adding or subtracting numerators and denominators separately. | Procedural | i. Confusing the rules of multiplication of fractions to the rules of division.  ii. Treating numerators and denominators as independent numbers |
| Percentages | Changing Percentages into decimal or fractional form | Multiplied by 100 instead of dividing by 100 | Conceptual | Considering the term ‘Percent’ to mean 100 parts instead of th part |
| Whole Numbers | Adding and subtracting of whole numbers using the vertical method | 1. Making incorrect alignment of digits 2. Skipping ‘carrying figures’ 3. Interchanging the arrangement of numbers for easier computation | Procedural | i. Does not know the importance of aligning digits as per respective place values  ii. Carelessness mistake  iii. Confusing the rule of commutative in addition to apply it on subtraction |
| Multiplication and Division of whole numbers by using long methods. | 1. Obtaining incorrect products of digits in top and bottom numbers 2. Skipping carrying figures. 3. Borrowing twice when required to do it once 4. Bringing down two digits at once in the algorithm of long division method. | Procedural | i. Had no clear understanding of multiplication tables  ii. Carelessness  iii. Thought that borrowing is done continuously  iv. Did not know the idea of a number going ‘zero times’ into another number. |
| Mixed Operations | Computation of mathematical expression involving more than one of the arithmetic operations. | Computing from left to right irrespective of the BODMAS rule. | Procedural | Had no clear understanding of the BODMAS rule. |
| Subtraction of integers | Computation involving integers and  ‘a minus sign’ | 1. Considered both ‘a negative sign’ and a ‘minus/subtraction sign’ to mean the same thing 2. Omission of ‘a negative sign’ 3. Making incorrect generalizations of the rules for addition and Multiplication of integers. | Conceptual | i. Confusing a negative sign from a minus sign  ii. Confusing the rules of multiplication to the rules of  addition. |

**4.7 Remedial Approach**

In objective five the study sought to develop an appropriate remedial approach for addressing the noted errors and misconceptions committed by Form I entrants in Tanzania. Subsection 4.7.1 presents the suggested lesson format, and section 4.7.2 presents a sample of a remedial lesson.

**4.7.1 Effective Format of Remedial Lesson**

This designed format has eleven crucial steps as developed from Dowker (2004) and other sited literature in chapter two. The steps are as follows: (i) Formulating remedial lesson objectives, (ii) Preparing/identifying the required conceptual knowledge, rules and procedures, (iii) Organizing teaching aids, (iv) Devising group activities, (v) Delivering remedial instructions, (vi) Providing worked examples, (vii) Providing group activities, (viii) Summarizing main points, (ix) Giving homework, (x) Doing assessment and evaluation, and (xi) Getting feedback as well as making a reflection.

**i. Formulation of Remedial Lesson Objectives**

A remedial teacher is expected to prepare some teaching objectives which are easy to achieve, and those for which students will acquire the desired knowledge after the completion of a topic. The objective of a lesson may be that, the students will be able to do something at the end of the lesson that they could not do before. As recommended by Suffolk (2004), good lesson objectives should among other things encourage students to think for themselves, to express themselves, and to improve their problem solving ability.

**ii. Identification of Conceptual Knowledge, Rules and Procedures**

The remedial teacher should use textbooks as the main source of material. This is because the textbooks contain the important definitions, rules, procedures, properties, notations, and conventions required for the course (Suffolk, 2004). However, the teaching should not be directed by a textbook since there is no need to cover all the content in it. For this reason, the remedial teacher is required to select only the appropriate learning materials that are in line with the remediation objectives. It is also advised that, when extra materials are needed, the teacher should utilize supplementary books and other published materials. Remedial materials selected from a textbook, and supplementary books, should be compiled by the teacher for use. Since the prime aim of the course is to address areas in which students have learning difficulties, the prepared materials should be simple, interesting and enjoyable to enhance students’ effective learning.

**iii. Organizing Teaching Aids**

Teaching aids are very important in the teaching of mathematics. The main function of teaching aids in mathematics is to bridge mathematics concepts that are abstract in order to make the student to understand the meaning of the concept. In this way students are motivated to learn. Busbridge and Womack (1991) describe a good teaching aid as something which can be used by the teacher and pupils to demonstrate or explain a mathematical idea. As such, writing chalks and charts are not ideal teaching aids. But these can be manipulated in some ways to become teaching aids. Also in order to foster an effective remedial teaching, the teacher is expected to identify learning areas which need the use of teaching aid, determine teaching aid that fit students’ cognitive level and practicing how to use the teaching aid in a simple way.

**iv. Learning Activities**

The remedial teacher should devise various learning activities basing on the remediation objectives. There should be different learning activities with the same teaching objective so as to develop students’ varied abilities, and skills in comprehending the desired knowledge. It is more effective for teachers to adopt a series of relevant and simple teaching activities than assigning one long activity, since students may acquire the desired knowledge and skills through diversified activities. Through these activities the teacher should be able to check whether the learning objectives have been accomplished.

Milkova (2000) offers five questions to consider when designing learning activities: (i) What will I do to explain the concept? (ii) What will I do to illustrate the concept in a different way? (iii) How can I engage students in the lesson? (iv) What are some relevant real-life examples, analogies, or situations that can help students understand the concept? (v) What will students need to do to help them understand the concept better?

Xavier (2007) urges that, when activities are well planned and carried out, they give pleasure of learning and develops self-reliance in students, and that way, the students are gradually encouraged to learn new concepts and make the learning in a joyful manner.

**v**. **Delivering Remedial Instructions**

Presentation and explanation of learning instructions requires the teacher to ensure that all students have developed a full understand. For this to build up, it is more desirable for students to know what they will be learning and doing during the lesson. A remedial teacher can therefore write the learning objectives on the board for the class. This will help students to follow the lesson presentation and understand the basis of class activities.

“How to impart mathematical knowledge?” and “How to enable the student to learn it?” are the two most important questions to be answered by the teacher before commencement of teaching. There are two main methods of teaching mathematics, namely Inductive and Deductive methods. Usually, a combination of the two methods in teaching mathematics is recommended. According to Prakash (2011), teaching of mathematics should be started with inductive method, and should end in deductive method. Describing the two methods separately, Prakash (2011) says that, deduction is a method for which, the teaching proceed from particular to general, from concrete to abstract, from known to unknown, and from special example to general formula.

This combination of methods facilitates the construction of formula with the help of sufficient number of concrete examples. Deduction method is described as the opposite of inductive method. With deduction method, the learner proceeds from general to particular, from abstract to concrete, and from formula to examples. As discussed by Prakash (2011), when using deduction method, the teacher should explain the application of the formulae to problems, and solve a number of such problems on the blackboard.

Through discussion on how to solve the problems, the students come to learn as to how the formulae can be applied. The remedial teacher is therefore urged to use a combination of the two methods with a provision of group activities. While students working in their groups (normally comprising of 4 to 6 members), the teacher should move from group to group giving assistance and encouragement, and asking thoughts provoking questions as the need arises.

**vi. Worked Examples**

A worked example is a step-by–step worked out problem. This is usually done intentionally to show students how particular problems are solved. In the beginning, the teacher show a fully worked out example, the next problem is worked out except for the last step, the second problem is worked out except for the last two steps. According to Busbridge and Womack (1991), worked examples enable learners to abstract a concept from the learning situation. A remedial teacher should carefully choose examples to illustrate each key point of the lesson. It is also important for the teacher to check that the examples are graded from easy to difficult, and that simple examples are discussed first with students before the hard ones.

**vii. Group Activities**

Activity based teaching is another technique used for teaching mathematics. One advantage of the class activities is that, the students are made active during the lesson. The teacher should provide the appropriate learning activities for each concept of the subject, and the given activity should not make students to diverge from learning an intended concept (Xavier, 2007). It is stressed here that, the remedial teacher should devise different learning activities with the same teaching objective so as to develop varied abilities and skills amongst students in problem solving. When students work together in small groups to solve problems they, often ask more questions, get more answers and do more quality thinking than when they work quietly, alone.

Busbridge and Womack (1991) contends that, diversified teaching activities such as group discussion, group presentation, games, role play, recording, visit and experiments may help students improve their interest in the subject. Teachers should therefore encourage students to participate actively in the class activities that are organized in small groups. These activities help them to understand the conceptual knowledge, and master the skills of collaborative learning. Where possible remedial teachers are advised to train-up some students who perform better in the subject to become “little teachers” and who will be responsible for helping their classmates with learning difficulties in group teaching, and self study sessions as well as outside class.

**viii. Summarizing the Lesson**

Summarization is among the top nine most effective teaching strategies in the history of education (Friend, 2001).  After the remedial teacher has delivered the learning instructions, the next step is to make a summary of the lesson’s main points. Either the teacher may decide to state the main points of the lesson by himself, or he/she may ask students to jot down what they think were the main points of the lesson. The second option requires the teacher to review students’ responses for determining their understanding of the concepts, and elaborate whatever may appear unclear. It is also mostly desirable for the teachers to guide their students to link up the knowledge learned with their life experiences so as to enhance effective learning.

**ix. Providing Homework**

The homework is a tool for evaluation and feedback. To the students, homework increases the motivation and effectiveness of learning. Exercise available in the books should be selected carefully and adjusted for the homework basing on the remedial teaching objectives. As it is pointed out in Busbridge & Womack (1991), the form and content of homework should be of variety of questions so as to develop students’ creativity, self learning and collaborative skills. Students can do and check their homework as follows: half the class can do all questions of the odd numbers in the textbook. The other half can do questions of the even numbers. Then students in groups can check their answers and if necessary, do corrections. Any problems that cannot be solved or agreed upon, these can be given to another group as a challenge. Alternatively, the teacher may select a few examples that need to be checked. Then invite a different student to solve each example on the board, and explain it to the class. The students chosen should be those who did the examples correctly at home.

**x. Assessment and Evaluation**

Rusbult (2007) describes the concepts of Assessment and Evaluation as follows: assessment provides a feedback on knowledge, skills, attitudes, and work achievement for the purpose of elevating future performance, and learning outcomes. Evaluation determines the level of quality of a performance or outcome and enables decision-making based on the level of quality demonstrated. As such, these two processes are complementary and necessary in education. In order to determine the level of students’ understanding, the remedial teacher should prepare assessment tools when planning the lesson. Students’ assessment on acquisition of conceptual, procedural knowledge and skills can be done during and after the delivery of learning instructions. The useful assessment tools include quizzes, exercises, tests, presentation, and personal reflection.

**xi. Feedback and Reflection**

Rusbult (2007) describes feedback as the most significant instructional strategy to move students forward in their learning. Feedback also provides students with an understanding of what they are doing well, links to classroom learning, and gives specific input on how to reach the next step in the learning progression. Through feedback, the teacher is enabled to work upon on areas in which students still have learning difficulties, and hence to plan for further remediation action. As such, the remedial teacher should check constantly the performance of students when answering oral/written questions, doing presentations, and when doing personal reflection. This should go together with allowing students to make a reflection on the lesson.

**4.7.2 Sample Remedial Lesson**

This subsection presents a sample lesson for remedying errors that were committed by respondents in performing operations with negative integers. The presentation follows the stages described in subsection 4.7.1.

**Remediation of Errors on Computation of Negative Integers**

*Problem area*

Computations involving negative integer

*Common errors*

1. Considering a minus sign in the expression to have the same function as the negative sign when the two are in consecutive order, hence ignoring one of them.
2. Using the rules of multiplying integers in place of rules of adding them.

*Associated misconceptions*

1. Assigning incorrect meaning of a negative number proceeded with either a positive or a negative sign.
2. Confusing the rules of multiplication from the rules of addition

*Causes*

1. Students have no clear concepts of negative integers
2. Students do not know that, *a* – *b* and *–a – b* can be expressed by using a ‘plus sign’.



*Specific objectives*

By the end of this unit the teacher should be in a position to enable students to:

1. Describe a negative integer
2. Differentiate a negative sign from a minus sign
3. Tell the meaning of – (–a ) and – ( +a)



1. Identify different ways of writing sums and differences involving negative numbers.

*Teaching and learning materials*

1. Coloured chalks
2. Marker pens
3. Flip chart
4. Computer graphical displays

**Conceptual knowledge, rules and procedures**

All numbers that are *less than**zero* are called *negative numbers*. On the number line, negative numbers are to the left of zero. Numbers to the right of zero are positive numbers. Negative numbers are denoted by asymbol (–) written just in front of a digit. It is also important to note that the (–) sign placed between two numbers is a *minus sign.*When this sign is placed just in front of a number it is*a negative sign.*

Therefore;

2 + (–7) means 2 plus negative 7

–2 – 1 means negative 2 minus 1

5 – (–3) means 5 minus negative 3



Real-life examples can be of much help to illustrate the concept of negative numbers.

1. *Negative temperature*: When water is very cold, the temperature reading on the thermometer is close to zero. When water freezes to form ice, its temperature is below zero, hence negative reading of temperature.
2. *Comparison of quantities*: Brian scored 5 points *less than* Shedrack (–5), Meshack scored 2 points *more than* Fred (+2)
3. *Business:* In business, capital money is a zero point, profitmoney is positive, *loss*money is negative.
4. *Language:* In common language, the positive and negative terms can be indicated by directional words:

*Negative**Positive*

BelowAbove

Down Up

Poor Rich

Short Tall

For each pair, there must be a reference point. This reference is the zero point as for the number line.

**Meaning of –*a*,** – **(+*a*) and – (**– ***a*)**



‘Negative’ is the opposite of ‘Positive’ and vice versa. Therefore, negative two (–2) is the opposite of positive two (+2) and vice versa.

For the same reason – (–2) means the opposite of –2 which is +2

Since the man with 0 shillings is better off that the one billed 50/=, it means that 0 is greater than –50. On the number line, –50 is on the left of 0. This fact helps us to

generate a rule that:

*if –a is on the left of –b then –a –b -a -b 0*



**Activity 1**

|  |
| --- |
| Ask students in pairs to write/state the following in words   1. 5 + 7 2. 7 – 3 3. 34 – (–10)  1. 8 – (+4) 2. 45 + (–1)  1. – 9 – 3   Assist them to write them in correct way |

**Activity 2**

|  |
| --- |
| Guide students to write the values of the following by using the concept of ‘opposite number’.   1. – (2) 2. – (– 26)  1. – ( +5)   Assist students to write the correct values. |

**Subtraction of negative numbers**

Subtraction of negative numbers is difficult to most students. This is because students are given just rules to apply without knowing how the ruleswere developed**.** Forexample (–) × (–) = (+); (–) × (+) = (–); and (+) × (–) = (–). Misunderstanding of these rules causes most students to get the same answer for the simplification of *a* – *a* , –*a* – *a* and *–a – (*–*a )*



The number line is commonly used to describe the concept of addition and subtraction of integers. Students should be reminded to observe the following rules:

1. Adding a positive number …move right

*a* *a* + 2

1. Subtracting a positive number… move left

*a* 2 *a*



iii. Adding a negative number… move left

*a* + (2) *a*



iv. Subtracting a negative number… move right

*a*  *a \_ (-2)*

**Example 1**: Add on the number line

i. 1+ (–3) ii. –4 +3



**Solution**

1+ (–3) means to move 3 steps to the left of 1 and 4 +3 means to move 3 steps to the right of 4.



1 + (3)



-4 -3 -2 -1 0 1 2 3 4

4 + 3 = 1



**Activity**

|  |
| --- |
| Guide students in small groups to simplify –2 – (–2) and 2 – (+2) by using   1. The concept of using a negative sign outside brackets as ‘the opposite’ of the number inside the brackets 2. The number line   Assist them to conclude that:  *a* – (–*a*) = *a* + *a*and  *a* – ( +*a*) = *a* – *a* |

**Example**

Find –9 – 3

**Solution**

Write –9 – 3 as –9 + (– 3)



–9 – 3 = – 9 + (– 3)



= – 12

**OR**

Use the number line: Move 3 steps to the left of –9. The answer is –12

**-**12 -11 -10 -9 -8 -7 -6 -5 -4 -3 -2 -1 0

9 + (– 3) = 12



**Summary**

1. All numbers to the left of ‘Zero’ on the number line are called negative numbers
2. All numbers to the right of ‘Zero’ on the number line are called positive numbers
3. A symbol (–) placed just in front of a digit is a negative sign
4. A symbol (–) placed between two numbers is a minus sign
5. – (–5) means the opposite of –5



1. *m* – *m* ≠ –*m* – *m*

**Reflection**

Seek students’ views about the lesson. The following questions may help to capture their evaluation remarks on the lesson.

1. Did you enjoy the lesson?
2. Which area of the lesson was mostly understood to you?
3. Which area of the lesson needs more clarification?
4. Which area of the lesson was not understood to you?

**Exercise**

Find

1. 9 – 3
2. 6 – (+3)
3. –3 – (–3)
4. 0.89 – (8.9)



**4.8 Concluding Remarks**

This chapter presented the findings on the extent to which Form I entrants were competent in numeracy skills. The pattern emerging from the major findings shows that, the majority of respondents had no mathematical skills adequate for learning secondary mathematics, and no effort was made at school level to address the problem. A total of 33 types of errors were identified. These were either procedural or conceptual. Noted misconceptions were due to incomplete understanding of concepts, incorrect use of mathematical rules and false generalization. The chapter also, presents the designed remedial approach for removing the noted mathematical errors and underlying misconceptions committed by Form I entrants in Tanzania.

**CHAPTER FIVE**

**5.0 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS**

## 5.1 Introduction

The first section of this chapter presents a summary of the study. The second section gives the conclusions made based on the study findings. The last section gives recommendations and suggestions for further study.

## 5.2 Summary

This study sought to investigate the mathematical ability of Form I entrants and the types of errors they commit in numeric computations. More specifically, the study sought to identify the areas in the content of Primary school mathematics in which respondents failed to solve the given questions, errors committed by respondents, the associated misconceptions and their causes. The study also sought teachers’ opinions regarding the ability of Form I entrants in mathematics. Furthermore, the study developed a remedial approach for addressing the common errors revealed in this study.

Both, purposive and random sampling techniques were used in this study. Nine sample schools were purposively selected from four categories of secondary schools in the Eastern Inspectorate Zone. Respondents were randomly selected from each school. A descriptive design was employed with the use of elements of both qualitative and quantitative methods.

Three instruments were used in collecting data for the study. The main source of data was the MA-Test that was administered to Form I students in the nine secondary schools, during the second and third week after they had reported for admission. The test was used to determine both the difficulty levels of test items, and the types of common errors, and underlying misconceptions committed by the Form I entrants in solving the test items.

Supplementary data was collected from subject teachers through a structured questionnaire. A focus group-discussion guide was used during discussion sessions with selected low and average achievers in the administered MA-Test. Data was analyzed both qualitatively and quantitatively. Analysis of the data from the test was carried out using the PSSP software to determine frequency counts and percentages.

The major findings of the study as based on the research questions, are as follows: Regarding the first research objective which sought to find out the ability of Form I entrants in mathematics, it was revealed that over 66% of the students involved in this study had performed poorly in the PSLE- Mathematics subject. The assessment of teachers who teach mathematics in the sample schools was that, most Form I entrants did not have the necessary skills for learning secondary mathematics. As revealed by the teachers, there are no efforts made by subject teachers or the school at large, to identify the mathematical weaknesses of Form I entrants.

The second research objective aimed to identify the computational errors made by Form I entrants in numeric skills, and the associated misconceptions. The results of the administered MA-Test reveal difficulty levels of mathematical concepts and skills as follows: the most poorly answered questions were on conversion of units of measurements and estimation of quantities, for which 84% to 71% of the respondents failed to give correct answers. Next were questions on percentages, angles, algebra, number patterns and arithmetic operations involving negative numbers. These registered failure rates ranging from 70% to 50%.

Thirdly, were questions on the BODMAS rule, arithmetic computations with integers, fractions, mixed numbers and decimals, for which the failure rate ranged from 50% to 25%. Questions on addition of mixed numbers, fractions, decimal numbers, whole numbers and division of proper fractions were the least difficult having failure rates below 25%. Both conceptual and procedural errors were identified in the respondents’ working sheets. These were manifested in the use of wrong computations and application of inappropriate rules. However, carelessness mistakes were also found even though minimal.

The third and fourth objectives of this study sought to conjecture and to test the possible misconceptions and underlying causes for the ultimate aim of identifying the root causes of the misconceptions. Results of the focus group discussions showed that, most of the misconceptions which led to the identified errors were due to several factors including the problem of poor understanding of the subject content, confusion of the rules and procedures, as well as little practice in the use of mathematical rules and methods.

The fifth objective aimed to design an effective format of the remedial approach to be used in clearing the identified misconceptions. Moreover, a sample of a class lesson have been developed basing on the proposed format. The sample lesson has been prepared in such a way that it can assist teachers in areas where students have problems of understanding certain concepts, rules or procedures. Basically, the lesson format includes the following: common errors committed by students, associated misconception(s) and their causes, and remedial activities. Included also, are short notes on concepts, rules and procedures in connection with each lesson. Worked examples and exercises are included to stimulate further learning.

**5.3 Conclusions**

Based on the findings of this study, the following conclusions are made. Firstly, it can be concluded that, most primary school leavers joining Form I in secondary schools, lack some conceptual understanding and skills necessary for learning secondary mathematics. Despite this problem, very little is done at the school level if any, to diagnose the learning difficulties among Form I entrants for the purpose of taking intervention action. Consequently, teaching of secondary school mathematics is ineffective since students are lacking basic concepts and mathematical skills which are prerequisites for learning secondary mathematics. Secondly, we can conclude that students had difficulties in applying arithmetic rules and procedures when performing addition, subtraction, multiplication and division of numbers. This is a big problem as without a good mastery of the rules and procedures, learning of other topics in secondary school mathematics will be extremely difficult.

Most errors identified in the respondents’ working were conceptual and procedural. These were due to poor mastery of concepts, and skills learned at the primary school level. Thus, it can be concluded that, poor mastery of concepts and computation skills is more likely to be the outcome of teaching techniques that encourages learning misconceptions. This in turn, led to errors in mathematical computations. Lastly, since some students involved in this study gave meaningless responses in the context of particular questions, it can be concluded that such responses reflect total lack of understanding of the question, or total ignorance of what is expected.

## 

## 5.4 Recommendations

### 5.4.1 Recommendations for Action

In view of the findings from the present study, it is recommended that:

1. Students selected to join Form I in all secondary schools in the country, should be tested at the school level on their competencies in numeracy knowledge, and skills that are necessary for learning secondary mathematics. This will alert teachers on the areas that students have mathematical misconceptions.
2. All secondary schools should conduct remedial classes to Form I entrants, targeting to address all areas that students have mathematical misconceptions. It is very significant for subject teachers to ensure that, the misconceptions are resolved before students are subjected to secondary school mathematics, which demands for a good mastery of mathematical concepts and skills learned at the primary school level. Since remedial teaching is more than re-teaching what was not understood, school owners/heads of schools should empower subject teachers by retraining them on the effective ways of undertaking remedial programmes either through seminars or workshops or inviting subject experts.
3. Since researchers agree that misconceptions are picked throughout during learning, and that most misconceptions are difficult to overcome at once, this study recommends enough work and examples to be given to students during and after classes. In this regard, teachers should be encouraged to practice diagnostic approach, as well as applying teaching strategies which allow remedial activities to be carried out during normal class lessons.
4. Mathematics teachers should pay more attention to questions assigned to students by doing immediately scoring, identifying particular errors committed, and providing feedback through clear corrections. If this action is properly done, possibilities are there to help students to overcome most learning difficulties they may have.
5. The remedial approach developed in this study is just an intervention measure and not a permanent solution of the problem. The ministry responsible with education should review its policies on teacher training and textbooks to address the issues of teaching and learning mathematics at primary and secondary school levels.

### 5.4.2 Recommendation for Further Study

1. The present study should be replicated for pupils at the primary school level, preferably in Standard VI so that, identified errors can be remedied before the pupils complete primary school education.

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**APPENDICES**

**Appendix A:** **Mathematics Achievement Test**

Muda: Masaa 2

*Time: 2 Hours*

**Jina la shule: Namba yako………**

*Name of your school: Your Index No……*

**Jina la shule ya Msingi uliyotoka:………………Wilaya……………**

*Name of your former Primary school:District…………..*

**Maelezo/***Instructions*

1. Karatasi hii ina kurasa 3 zenye maswali 50. Jibu maswali yote.

*This paper has 3 printed pages with 50 questions. Answer all questions.*

1. Onesha njia uliyotumia kupata jibu kwa kila swali.

*Show your working for each question.*

1. Andika jibu kwenye nafasi iliyotolewa kwa kila swali.

*For each question write the answer in the space provided*

|  |
| --- |
| 4.Usiandike jina lako/*Don’t write you name* |

| Na. | Maswali/*Questions* | Njia/*Working* | Jibu/*Answer* |
| --- | --- | --- | --- |
| 1 | Jumlisha/Add 856 + 9134 = |  |  |
| 2 | Jumlisha/Add 0.021 + 9.98 = |  |  |
| 3 | Jumlisha/Add  43.003 + 14.01 + 58.507= |  |  |
| 4 | Jumlisha/Add 51/4 + 7 2/3 = |  |  |
| 5 | Jumlisha/Add 4012 + 679 = |  |  |
| 6 | Toa/ Subtract 2.11 − 1.173 = |  |  |
| 7 | Toa/Subtract 0 − 8 = |  |  |
| 8 | Toa/Subtract 97 − 971 = |  |  |
| 9 | Toa/Subtract 55/9  ─ 6 2/3  = |  |  |
| 10 | Toa/Subtract ─ 9 − 3 = |  |  |
| 11 | Toa/Subtract 33⅔ − 17⅓ = |  |  |
| 12 | Toa/Subtract 7.03 − 1.174 = |  |  |
| 13 | Toa/Subtract 0.89 − (− 8.9) = |  |  |
| 14 | Gawanya/Divide 9113 ÷ 13 = |  |  |
| 15 | Gawanya/Divide 7.65 ÷ 0.034 = |  |  |
| 16 | Gawanya/Divide 4 3/8 ÷ 21/2 = |  |  |
| 17 | Gawanya/Divide 15/28 ÷ 9/24 = |  |  |
| 18 | Gawanya/Divide 0.12 ÷ 0.003 = |  |  |
| 19 | Zidisha/Multiply 103/4 × 5/8 = |  |  |
| 20 | Zidisha/Multiply 0.29 × 1000 = |  |  |
| 21 | Zidisha/Multiply 58 × 347 = |  |  |
| 22 | Zidisha/Multiply 3.5 × 1¾ = |  |  |
| 23 | Zidisha/Multiply 2.14 × 0.029 = |  |  |
| 24 | Kokotoa/Calculate 6 − 8 ÷ 2 = |  |  |
| 25 | Kokotoa/Calculate  12 **─**  10 × 7 = |  |  |
| 26 | Gawanya/Divide 1/2 ÷ 1/6 = |  |  |
| 27 | Toa/Subtract **─** 6 **─** ( + 3 ) = |  |  |
| 28 | Gawanya/Divide  ( + 78 ) ÷ ( **─** 13 ) |  |  |
| 29 | Toa/Subtract ─ 3 ─ (3) = |  |  |
| 30 | Rahisisha/ Simplify −( −8 ÷ 4 ) = |  |  |
| 31 | Badili ¾ % kuwa sehemu rahisi.*Change into simple fraction* |  |  |
| 32 | Badili 2.6 kuwa asilimia.*Change 2.6 into percentage form* |  |  |
| 33 | Tafuta namba mraba ya 0.0473.*Find the square of* |  |  |
| 34 | Andika namba mbili zinazofuatana katika mtiririko huu: − 2, 4, 10, .*Write the next two numbers.* |  |  |
| 35 | Badili 12.25% kuwa desimali. *Change into decimal form.* |  |  |
| 36 | Tafuta wastani wa saa 1:40, saa 2:55, saa 8:05 na saa 3:20.*Find the average* *of* |  |  |
| 37 | Iwapom = − 2,n = 3 tafuta thamani ya m2 + mn – 2  *Evaluate m2 + mn – 2* |  |  |
| 38 | Km M Sm  3 25 75  × 5 |  |  |
| 39 | Tafuta thamani ya x iwapo ukubwa wa pembe katika umbo la pembe tatu ni:  1/2 x0 – 15, 2x0 + 300 , na 150.*If these are three angles of a triangle find the value of x.* |  |  |
| 40 | Rahisisha /*Simplify*  3m **─** 3n + (−8n) |  |  |
| 41 | Ikiwa 5x **─** 1 = 2 + x, tafuta thamani ya x. *Solve for x* |  |  |
| 42 | Nafikiria namba, naizidisha na 3 na kuongeza 4, jawabu ni 1. Tafuta namba ninayofikiria. *I am thinking of a number when multiplied by 3 and adding 4 the answer is 1. What is the number?* |  |  |
| 43 | Kadiria uzito wa mtoto mchanga katika gramu. *Estimate the weight of a newly born baby in grammes* |  |  |
| 44 | Kadiria urefu wa mlango wa chumba cha darasa lako katika sentimeta. *Estimate the length of your classroom door in cm.* |  |  |
| 45 | Kadiria muda utakaotumia kutembea kutoka kwenye mlango wa chumba cha darasa lako hadi kwenye mlango wa ofisi ya mkuu wa shule. *How much time will you take to walk from your classroom to the School Head office.* |  |  |
| 46 | Badili m31000 katika lita*. Convert into litres* |  |  |
| 47 | Badili m2 100 katika sm2.*Convert into sm2* |  |  |
| 48 | Badili m310 katika sm3. *Convert into sm3* |  |  |
| 49 | Kwa kutumia rula na penseli chora kwa kukadiria pembe yenye ukubwa wa nyuzi 1350.*By using a ruler and a pencil construct angle 135*0 |  |  |
| 50 | Punguza mita 40 kwa 20%.*Reduce 40m by 20%* |  |  |

**Appendix B:** **Questionnaire for Teachers**

This questionnaire is designed to collect information from classroom teachers on specific areas of investigation. Kindly, respond to the following questions according to the given instructions. The information that you give will be treated strictly confidential and used only for the purpose of this research.

**Instructions:**

* + - 1. Please fill in the blanks where applicable
      2. Tick in the appropriate space where applicable

**Section A: Background information**

Region: ……………………District: School:

Sex: Male (………) Female (…….)

1. What is your Professional qualification?

Licensed (……) Diploma (……..) Graduate (………) Others (…….)

1. What subjects you studied at Teachers’ college/ University? (I) …… (ii)…… (ii)……..
2. What subject you like most?
3. What subjects you teach at this school? (i)…………… (ii)…………… (iii)………….
4. What is your teaching experience? …………….
5. What is your teaching load per week? ..…. Periods
6. In which Forms you teach mathematics? …………
7. Have you ever attended any In- Service training/Seminar/workshop about teaching and learning of Mathematics? Yes (…..) No (………)

**Section B: Ability of Form I Entrants in Mathematics**

1. Apart from this Form I intake how many other intakes you taught mathematics before?
2. Do Form I entrants possess necessary mathematical skills for learning Secondary school mathematics? Yes (……) No (……..)
3. If the answer for item 10 is No, what common problems facing them in learning mathematics?
4. ……………………………………………………………………………
5. ……………………………………………………………………………
6. ……………………………………………………………………………
7. …………………………………………………………………………
8. What proportion/percentage of Form I entrants had problems listed in item 11?

(i)…………….. (ii) …………….. (iii) ……………. (iv) ……………………

**Section C: Assessment on Ability of Form I Entrants in Mathematics**

1. Respond to the following items basing on your experience of teaching Form I mathematics class. Put a tick (√) for your choice.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **No.** | **Item** | **Strongly Agree** | **Agree** | **Not Agree** | **Strongly Disagree** | **No Opinion** |
| i. | Most of Form I entrants have negative attitude towards mathematics. |  |  |  |  |  |
| ii. | Most of Form I entrants have poor background in mathematics. |  |  |  |  |  |
| iii. | Most of Form I entrants have inadequate skills on arithmetic operations |  |  |  |  |  |
| iv. | Teachers have a habit of identifying mathematical misconceptions amongst Form I entrants. |  |  |  |  |  |
| v. | There is a special programme at school to help Form I entrants who are weak in mathematics. |  |  |  |  |  |
| vi. | Form I orientation course conducted in some schools are helpful in improving their mathematical skills. |  |  |  |  |  |
| vii. | Drilling students with revision questions can help them to improve their performance in mathematics. |  |  |  |  |  |
| viii. | It is necessary to orient Form I entrants on arithmetic skills before introducing them to secondary mathematics. |  |  |  |  |  |
| ix. | The Form I mathematics textbook is suitable for beginners of secondary mathematics. |  |  |  |  |  |

**Appendix C:** **Focus Group discussion guide for selected respondents**

**1. Basic knowledge on Units of measurements**

1. What instrument will you use to measure the volume of water:
2. What is the expansion form of 1m2 and 1m3?
3. Is 1m2 = 100cm3
4. Is it possible for a tea spoon to hold 1m3 of water? Give reason.
5. How much time it takes the seconds’ arrow of a clock to move from 7 mark to 8 mark on the face of a clock?
6. Is the weight of your mathematics text book exceeding 100gms?

**2. Relations of Percentages, Fractions and Decimals**

1. What is the meaning of one percentage?
2. Is 5 the same as 5 + ?



1. Is 5 the same as 5. 9 ?



1. Which is greater:
2. 0.5 or 50%
3. or 0.5



1. 0.75 or 0.175

**3. Facts on Subtraction of numbers**

1. What is the result of subtracting a large positive integer from a small one?
2. What is the result of subtracting a small positive number from the large one?
3. How do you arrange two large whole numbers when adding or subtracting them?

**4. Addition, subtraction, multiplication and division of fractions**

For each of the following state whether **correct** or **not correct** and why?

a. + = b. × = = c. ÷ = =



d. - = =



**5. Computations involving negative and minus signs**

1. Is the answer for 2 – 2 the same as answer for 2 + (- 2)? Give reason.
2. Is the answer for 2 – 3 the same as answer for 3 – 2? Give reason.
3. What is the number opposite to – 3?
4. A certain number *m* is the opposite of negative nine. Express *m* in symbolic form.

**6. Estimations**

1. What angle is made when a door is opened half way?
2. What is the approximate sum of 6 + 11/12 ?
3. Is the width of your thumb approximating 5mm?

**7. Algebraic expressions and equations**

1. Is it true that 2y – y = 2. Give reason
2. Is it true that 2y – y = 2y – 1y. Give reason
3. If y = –2, find the value of 7 – y
4. Is it correct to say that if 3k – 3 , then 3k = –2 – 3? Give reason.



**Appendix** **D: Documentary Guide for Respondents’ Performance in the PSLE or School- based Entrance Examination.**

1. Name of School: ……………......School Category…………………District…
2. Number of Respondents……..
3. Number of Mathematics teachers at school…………
4. Entrance Criterion: PSLE………School Based Examination………(Tick the appropriate)
5. Analyze the respondents’ Performance in PSLE/School-Based Examination and fill in the following table:

|  |  |
| --- | --- |
| **Range of Scores**  **(Out of 50)** | **Frequency** |
| 1-10 |  |
| 11- 20 |  |
| 21- 30 |  |
| 31- 40 |  |
| 41- 50 |  |

1. (a) Is the school conducting Pre-Form I Orientation Course Programme? Yes……No……..
2. If the answer is Yes, what is the content of course programme? (Tick the appropriate)
3. Review of Primary school mathematics ……………………..………
4. Topics from the syllabus of Form I Mathematics………….……..…..**Appendix E: Research Clearance Letter**

